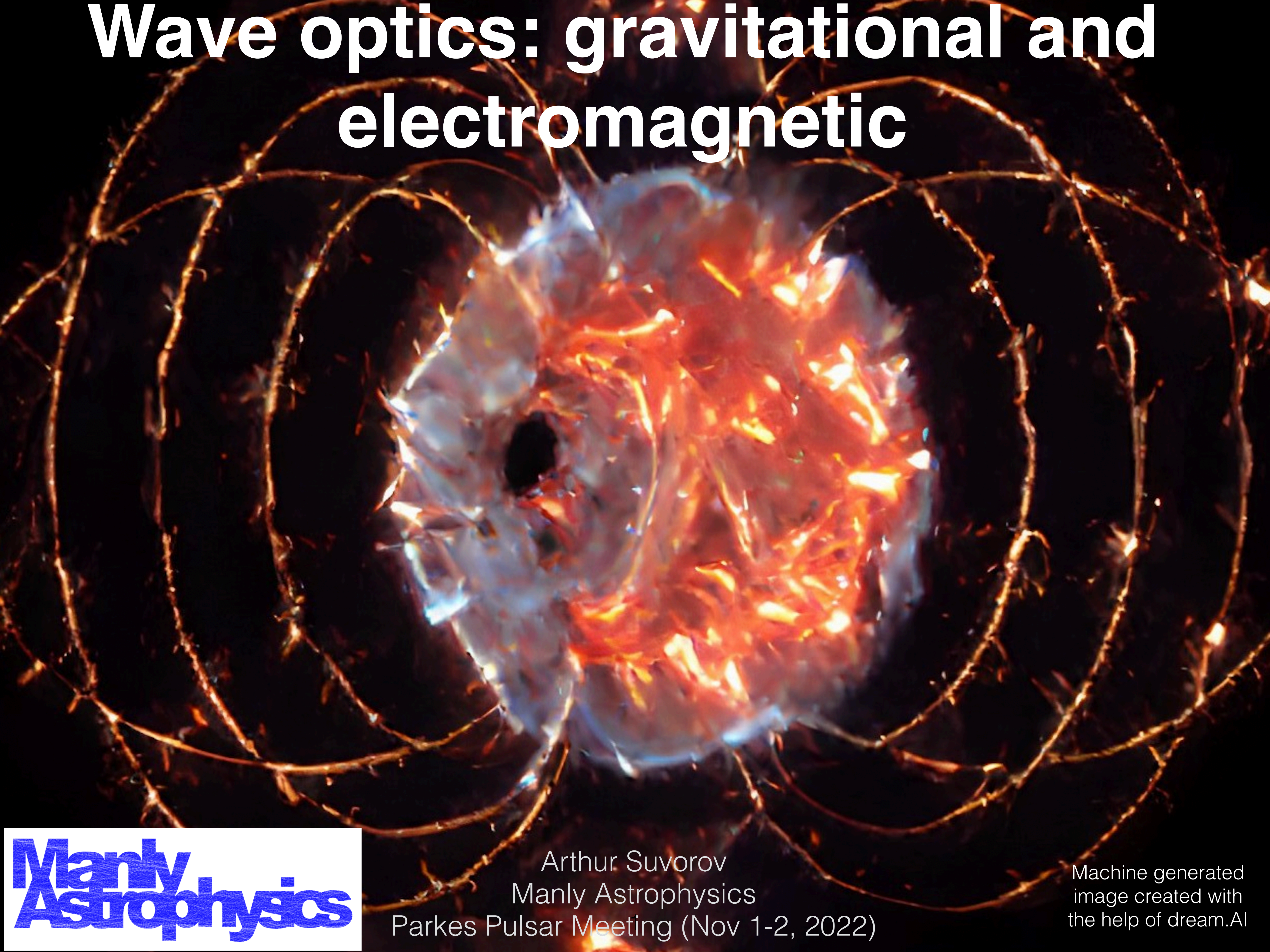
Wave optics: gravitational and electromagnetic



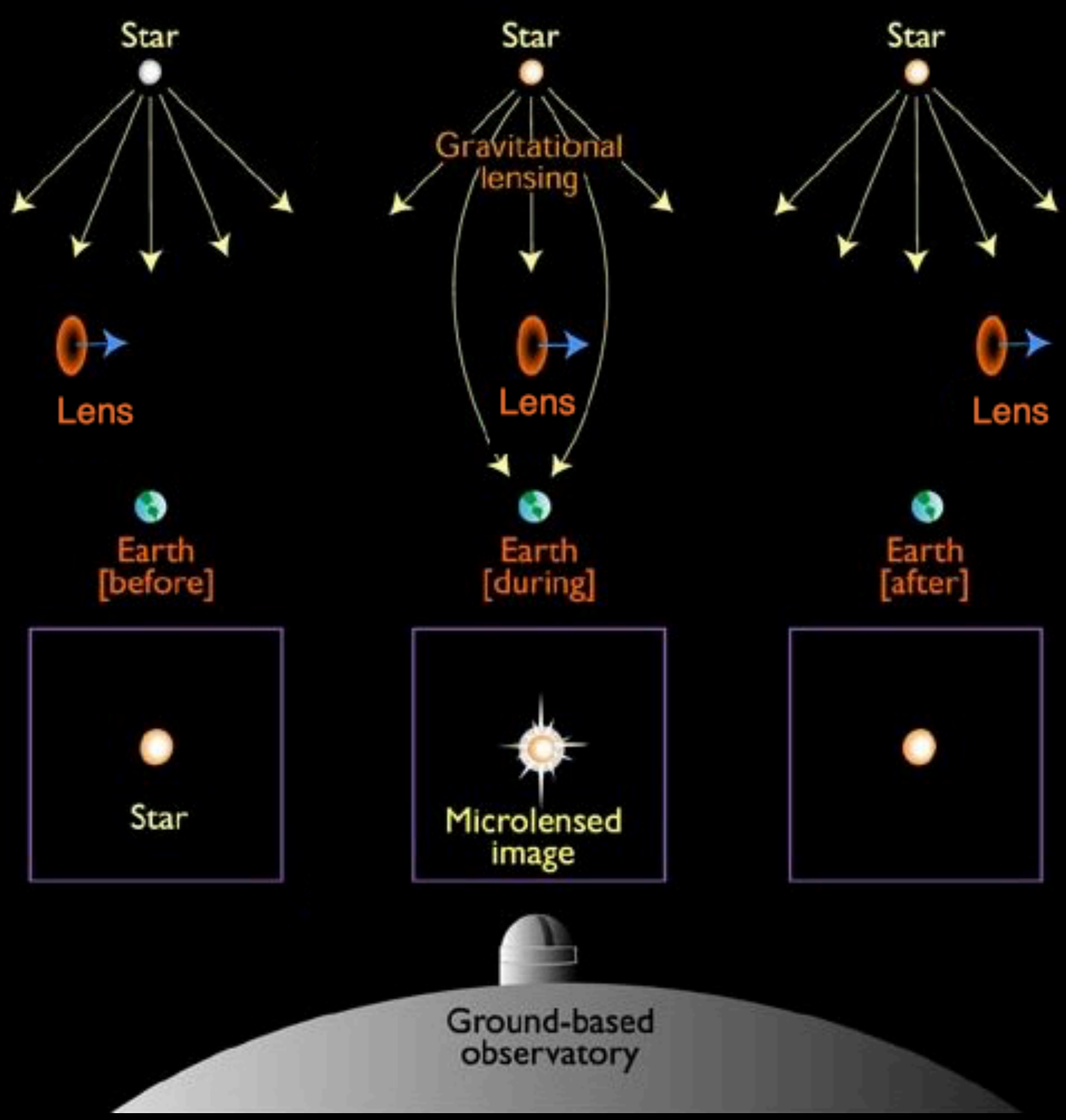
Manly Astrophysics

Arthur Suvorov
Manly Astrophysics
Parkes Pulsar Meeting (Nov 1-2, 2022)

Machine generated image created with the help of dream.AI

Basic lensing introduction

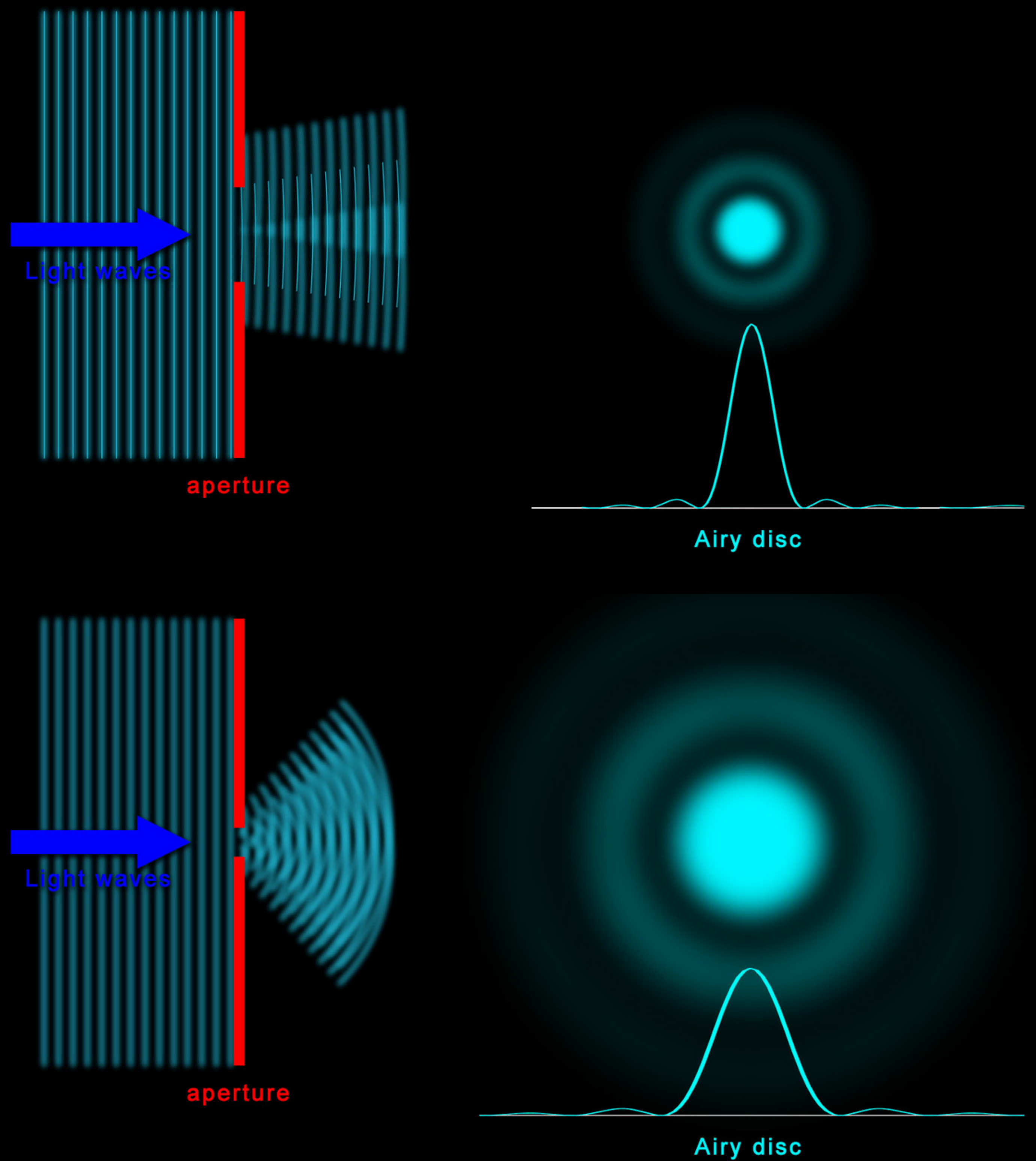
- Space is not empty. Radiation that reaches us from distant sources often interacts with intervening material, modulating the signal and possibly amplifying it. Can also result in multi-image formation.
- This "double-edged" behaviour can allow us to glean information about both source and line-of-sight material in some cases.
- For pulsars: we regularly receive signals that are lensed by the ISM or plasma, and gravitational waves might also be lensed by astrophysical masses (e.g., stars)



Modified from ESA/Hubble illustration [https://esahubble.org]

Long or short?

- If the radiation is short wavelength, then radiation packets can be treated as point particles traversing spacetime geodesics and can freely “pass through” the astrophysical slit.
- If, however, the wavelength is long relative to some characteristic scale, then significant diffraction takes place, and self-interference effects become important. More generally these regimes are not so clean cut, with some effects more important than others in each limit but often all important.
- The lensing of coherent, electromagnetic waves must generally be modelled in the wave optics regime (e.g., Melrose & Watson 2006)



Credit: Nando Harmsen, 2021.

Even with a model, what to do?

- Eventually, we want to evaluate integrals that look like this:

$$\int e^{i\phi} dx \sim \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz dx dy e^{i\nu t_d(\boldsymbol{x}_s, \boldsymbol{x})}$$

- “Path integral”: time delay contains geometric (i.e., triangulation) and lensing (e.g., Shapiro) terms

How?

- Numerical methods are few and far between, because:

#1: Euler’s formula: integrand is oscillatory like cos(x) + i sin(x); does not decay fast (cutoffs also fail).

#2: Conditionally convergent; certain methods involving coordinate transforms fail.

#3: Singularities in some cases (e.g., point-like stars in gravitational lensing case) ….

Picard-Lefschetz fundamental idea

- General strategy: complexity coordinates such that the oscillatory (imaginary) part of the integral is constant

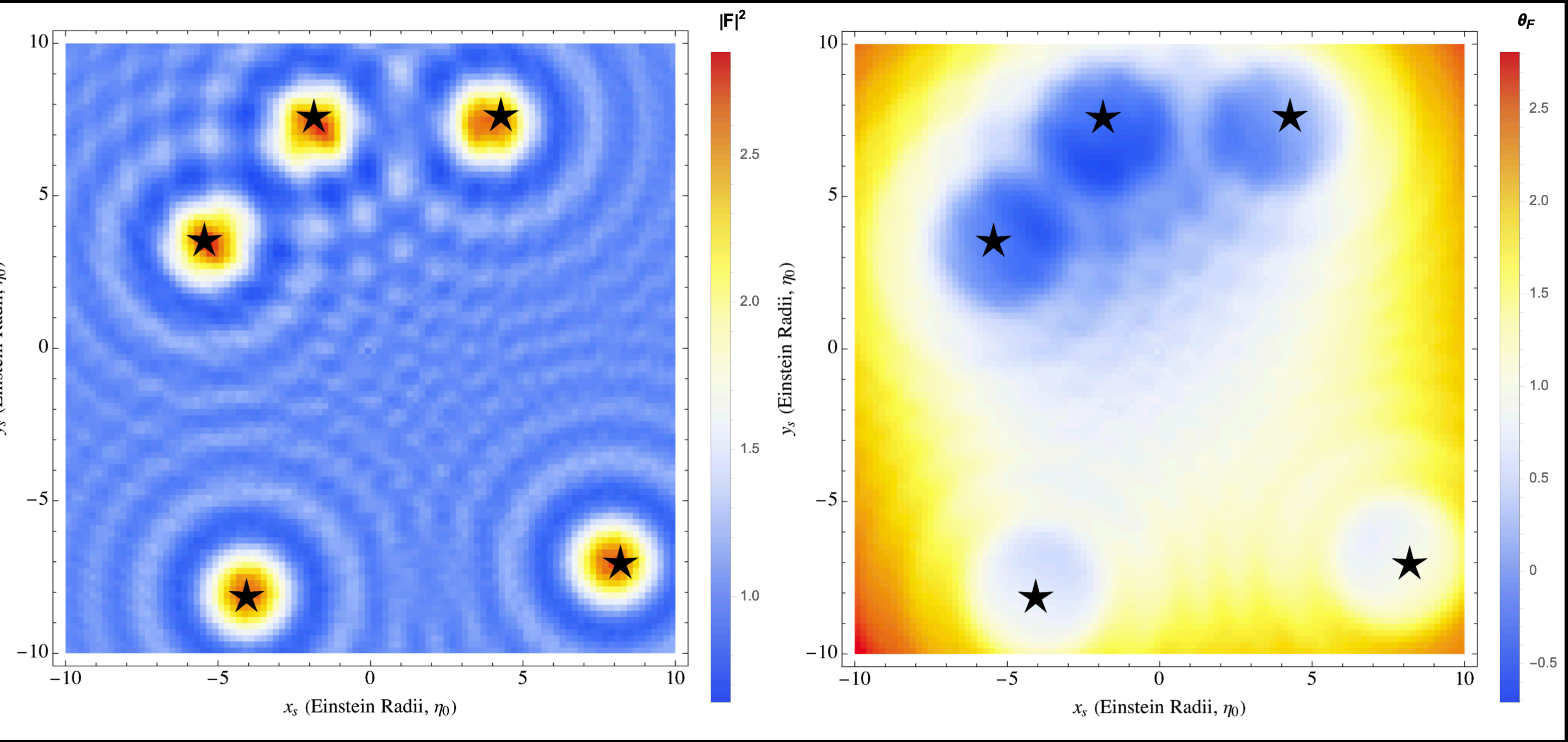
- In 1D: we have a phase and complexity x->z = x+iy

$$e^{i\phi(z)} = e^{i\phi(x+iy)} = e^{h(x,y)+iH(x,y)} = \overbrace{e^{h(x,y)}}^{\text{Well behaved}} \underbrace{e^{iH(x,y)}}_{\text{Oscillatory}}$$

If we manufacture a contour y = y(x) such that the function H is constant, then an integral along this curve is going to be well-behaved and evaluable with standard techniques

Refs: Witten 2010; Feldbrugge++2019;2020; Suvorov 2022

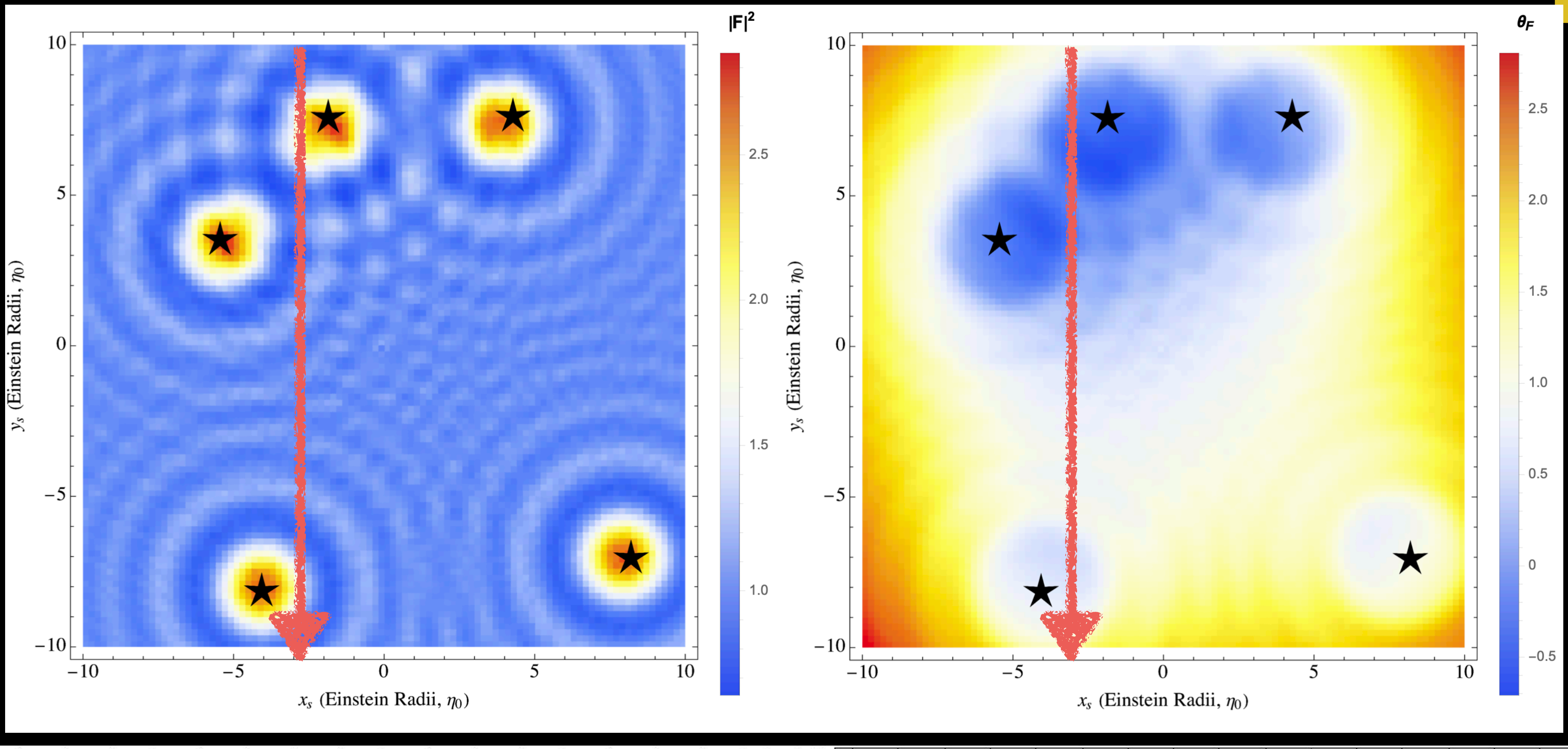
A simple GW example

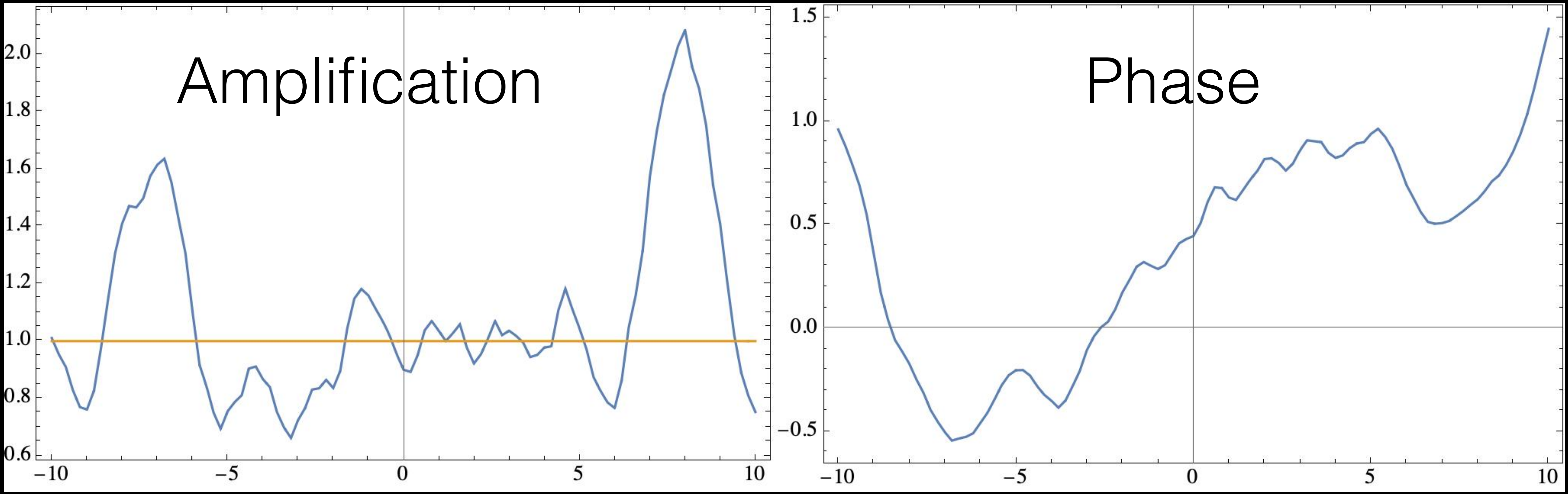


Here: five micro lenses of mass ~8 solars each, with a millisecond pulsar source (400 Hz GW frequency).

Relative motion: GW example

Suvorov (2022; ApJ)

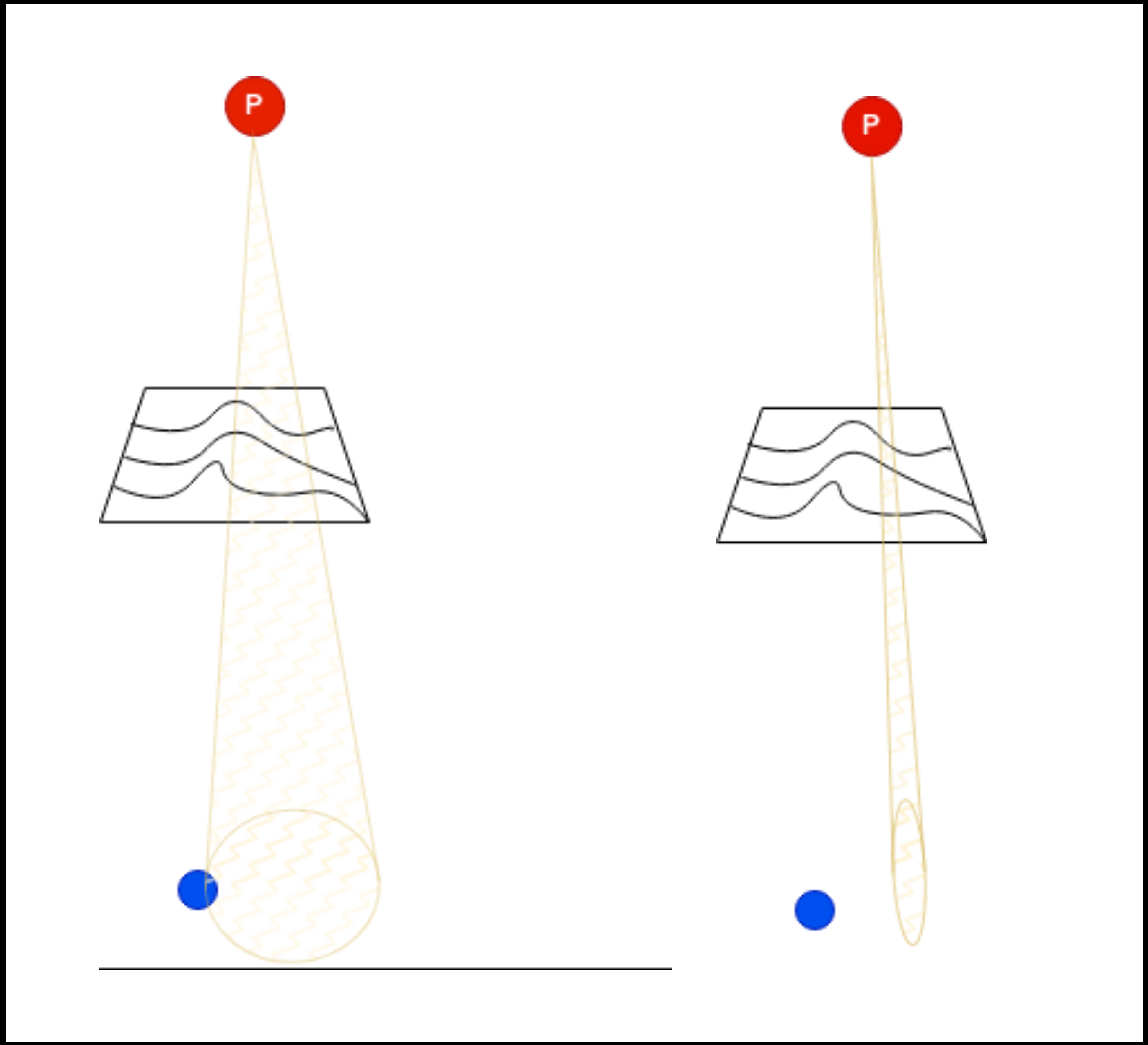




Imagine the source is moving relative to the lens over time: large phase variations could scramble phase-coherent nature of a search, and signal might be amplified or appear to oscillate

Plasma lensing: scintillation

- Two important sources for lensing of radio signals from pulsars, which require a wave-optics description, come from
- i) turbulence in the ISM (e.g., shear from galactic rotation), and
- ii) filamentary streams of (ionised) gas clouds (i.e., “snow clouds”)
- With first-principles and efficient calculations of the diffraction integrals, we may hope to learn something about the pulsar emission mechanism, because the diffraction pattern, witnessed by an observer, is described by the convolution of a point-source response with an image of the source. From the depth of modulation of scintillation, one can infer the size of the emission region (Gwinn et al. 2012; Johnson et al. 2012)



As put succinctly by Gwinn et al. (1997; ApJ 483 L53): “The modulation index of scintillation, m, provides a measure of source size: stars twinkle, planets do not.” $m^2 = \langle I^2 - \langle I\rangle^2\rangle/\langle I\rangle^2$

$$P_N(V) = \frac{2}{\pi}\frac{N^{N+1}}{(N-1)!}\frac{1}{(1-\rho^2)\kappa^2}\left(\frac{|V|}{\kappa}\right)^{N-1} K_{N-1} \times \left(\frac{2N}{1-\rho^2}\frac{|V|}{\kappa}\right)\exp\left(\frac{2N\rho}{1-\rho^2}\frac{\mathrm{Re}[V]}{\kappa}\right).$$

$$\kappa_\xi = \kappa_0(kM\theta\sigma_\xi)^2$$
$$\kappa_\eta = \kappa_0(kM\theta\sigma_\eta)^2$$

Example: turbulence in the ISM

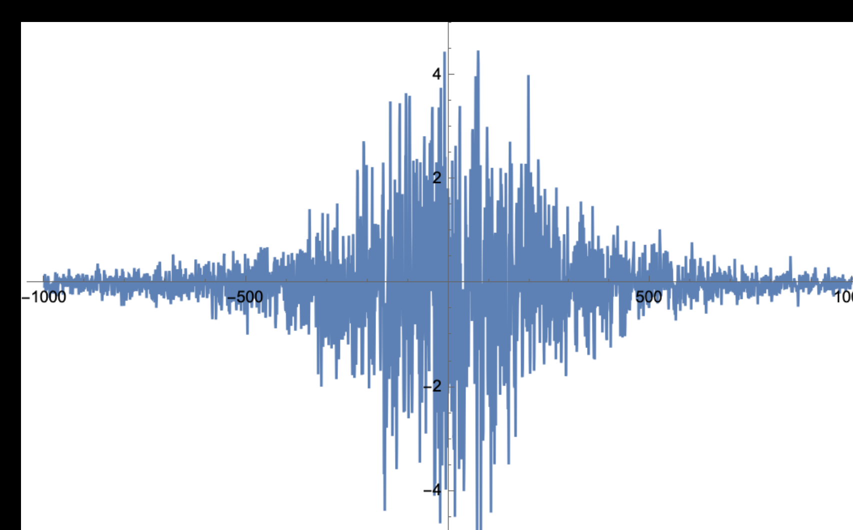
- If we assume that the the electron density power spectrum, Φ_{Ne}, follows a power law between some inner and outer scales, we can write the phase function as a random variable
- Some particular realisation "g", which is like Gaussian white noise, then defines the overall phase for a particular screen
- Popular example: Kolmogorov phase screen, β=11/3.
- In this case, Picard-Lefschetz can be used twice: one to actually define the phase, and then to integrate the overall profile to get the Fresnel-Kirchhoff integral.

$$t_{\mathrm{d}}(\mathbf{x}, \mathbf{X}) = \left[\frac{D_S(1+z_L)}{2cD_L D_{LS}}\left(\mathbf{x} - \frac{D_{LS}}{D_S}\mathbf{X}\right)^2 + \frac{\phi(\mathbf{x})}{2\pi\nu}\right]$$

$$\begin{aligned} P_{2\phi}(q_x, q_y) &= 2\pi z(\lambda r_{\mathrm{e}})^2\, P_{3\mathrm{N}}(q_x, q_y, q_z = 0) \\ &= 2\pi z(\lambda r_{\mathrm{e}})^2 C_N^2\left(\sqrt{q_x^2 + q_y^2}\right)^{-\beta}, \end{aligned}$$

Even the phase is hard to evaluate!

$$\phi(x, y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(q_x, q_y)\sqrt{P_{2\phi}(q_x, q_y)}\, \exp\left[-j2\pi(xq_x + yq_y)\right] dq_x dq_y,$$

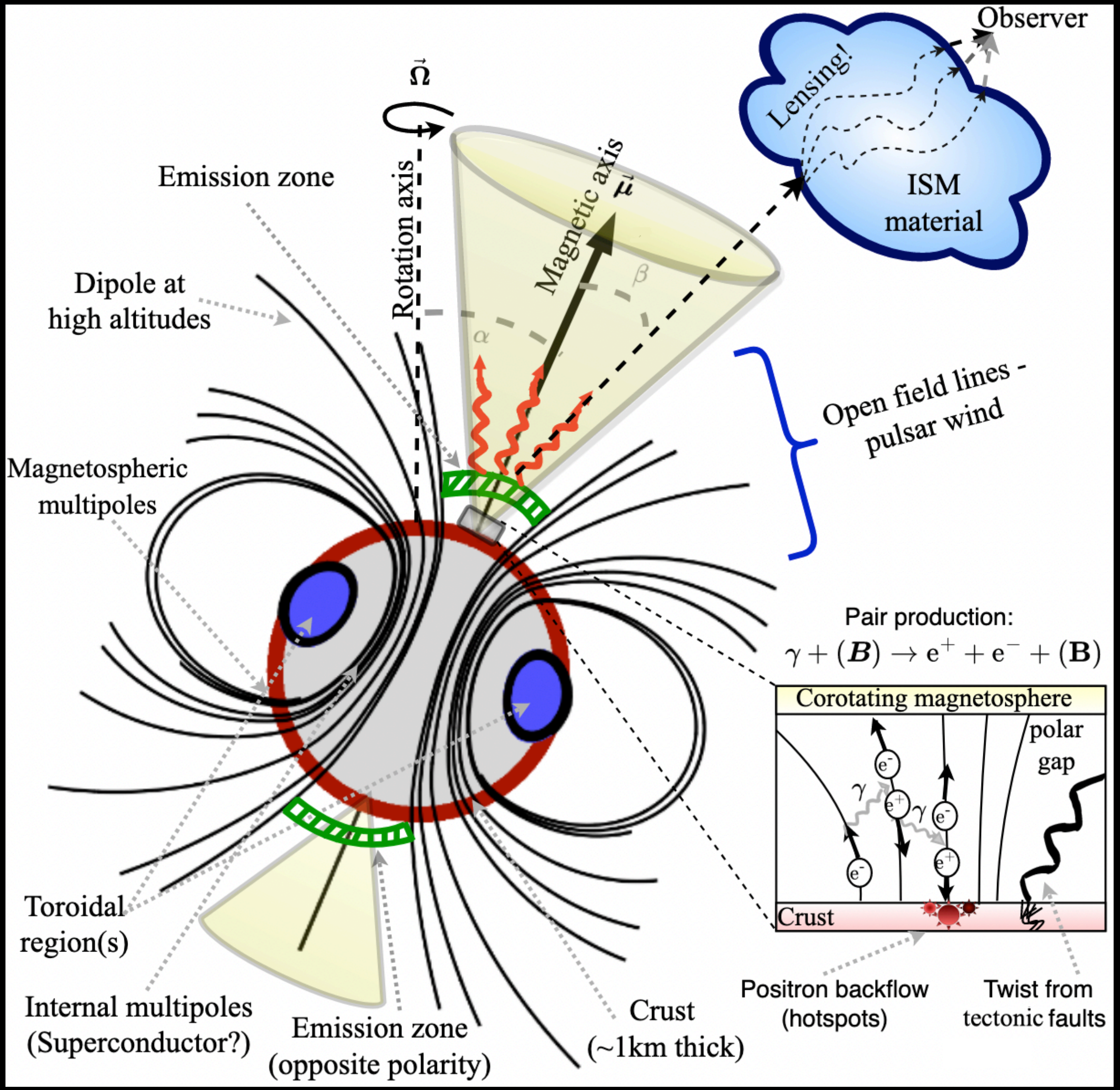


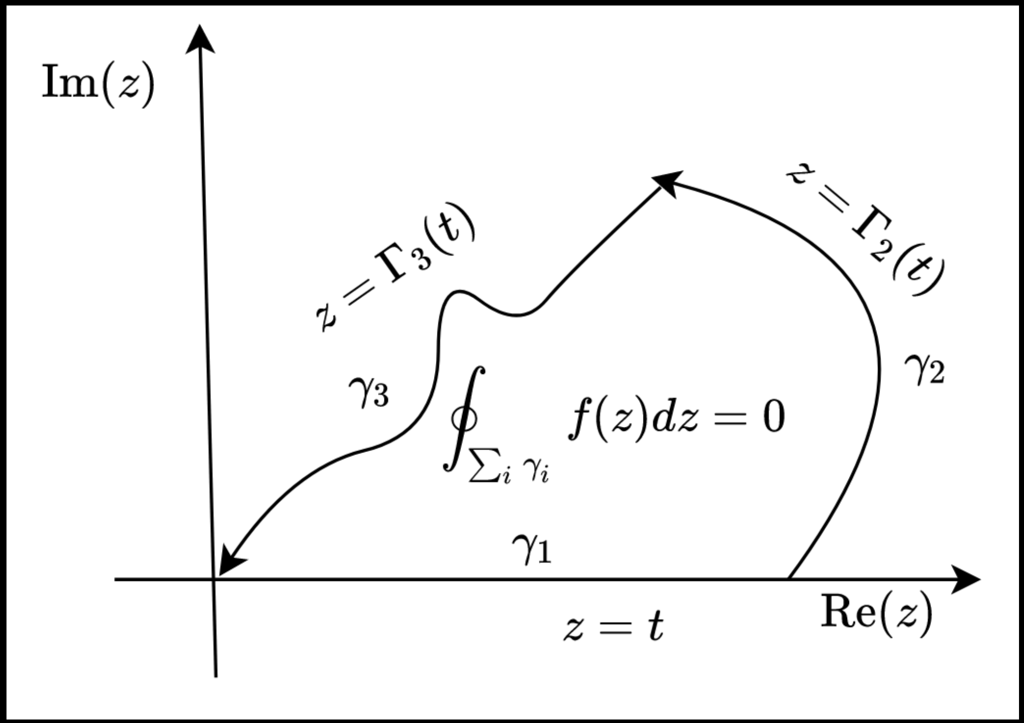
A particular realisation, projected onto the "qx" dimension

(See, e.g., Hamidouche & Lestrade 2007)

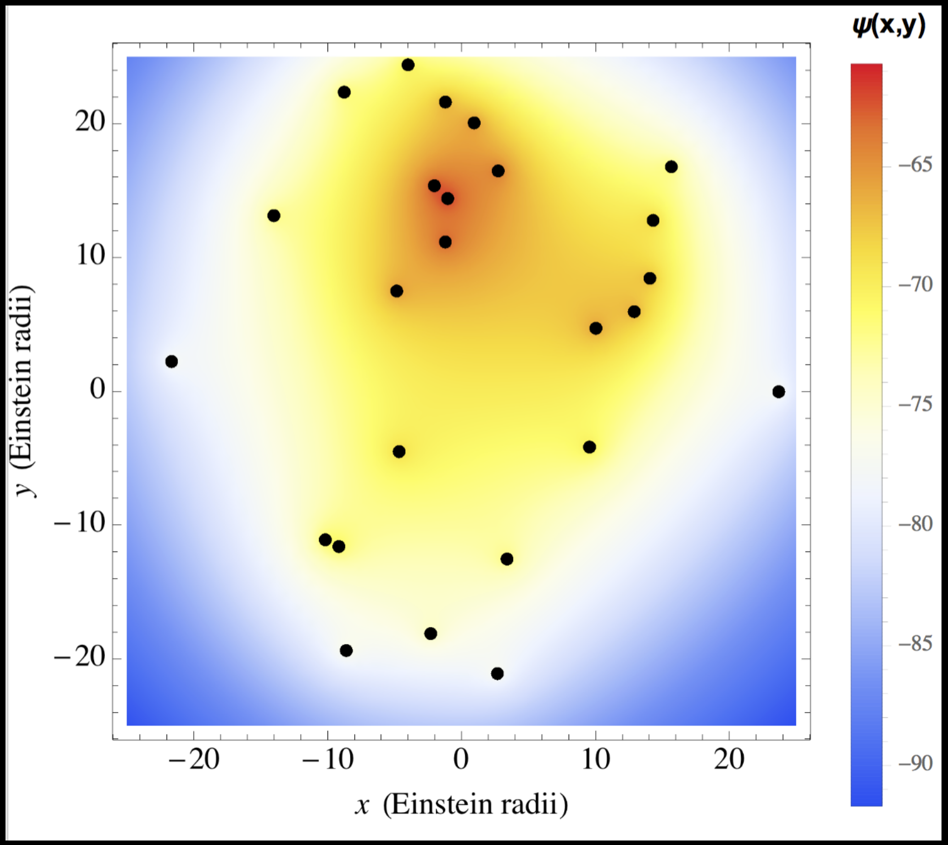
Summary

- Gravitational wave propagation, when encountering astrophysical objects, will tend to be lensed. Radio waves will be lensed by many forms of “stuff” in the ISM
- Importantly: the lensing will occur in the diffractive regime for GWs lensed by stars, or by coherent radio sources by plasma
- The calculation in this case is hard because of the oscillatory nature of the Fresnell-Kirchhoff diffraction integral.
- Using some ideas from Picard-Lefschetz theory, we make progress (ask if you want tools!)
- For GWs from NS in globular clusters: may need to worry about phase scrambling.
- For radio emissions from pulsars: may be able to learn something about emission geometry from first-principles calculations.

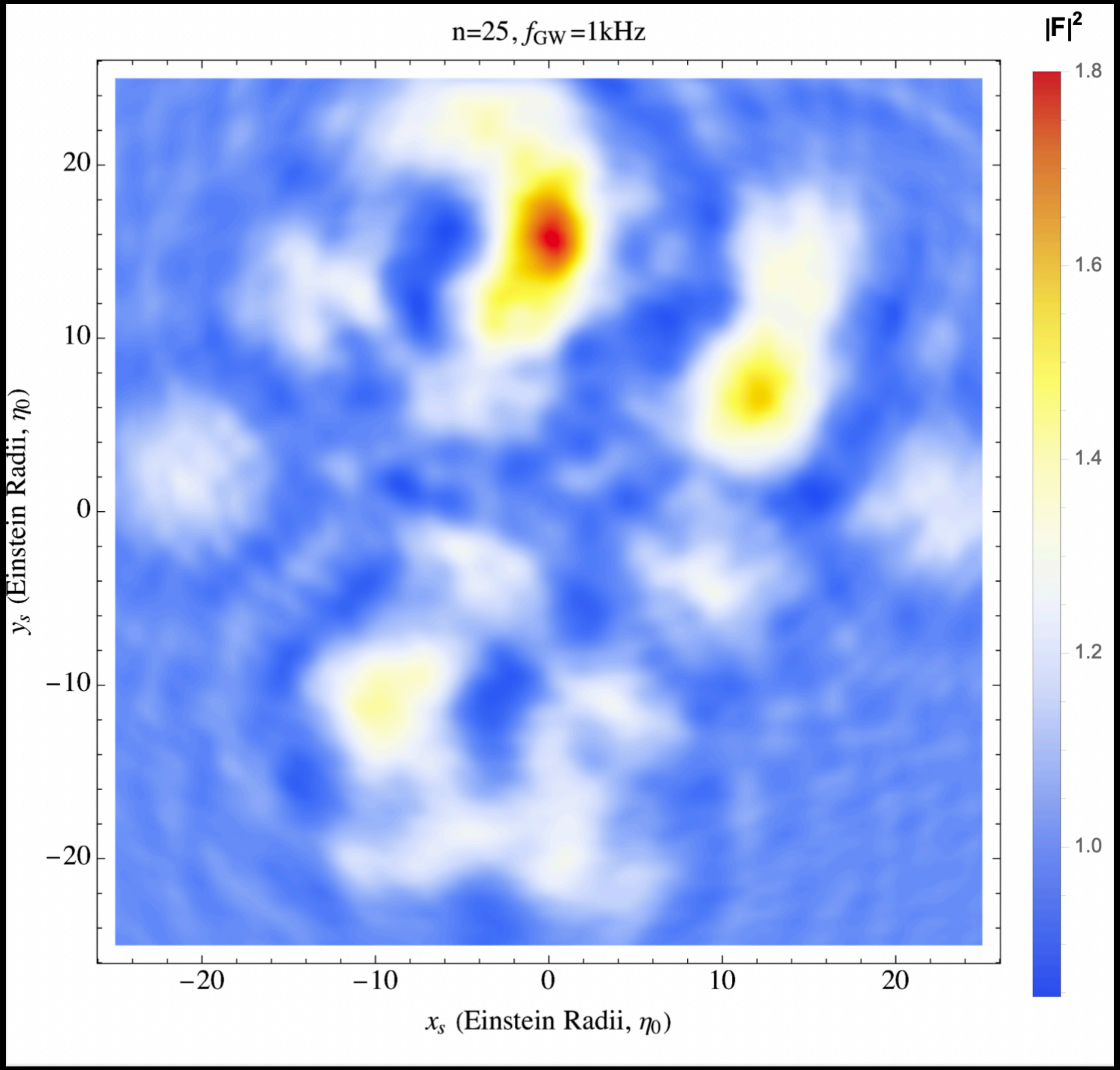




More complicated!



Depending on the relative motion of the pulsar through the plane, different 1D patterns emerge for the fluctuations in h(t)



Suvorov (2022; ApJ)

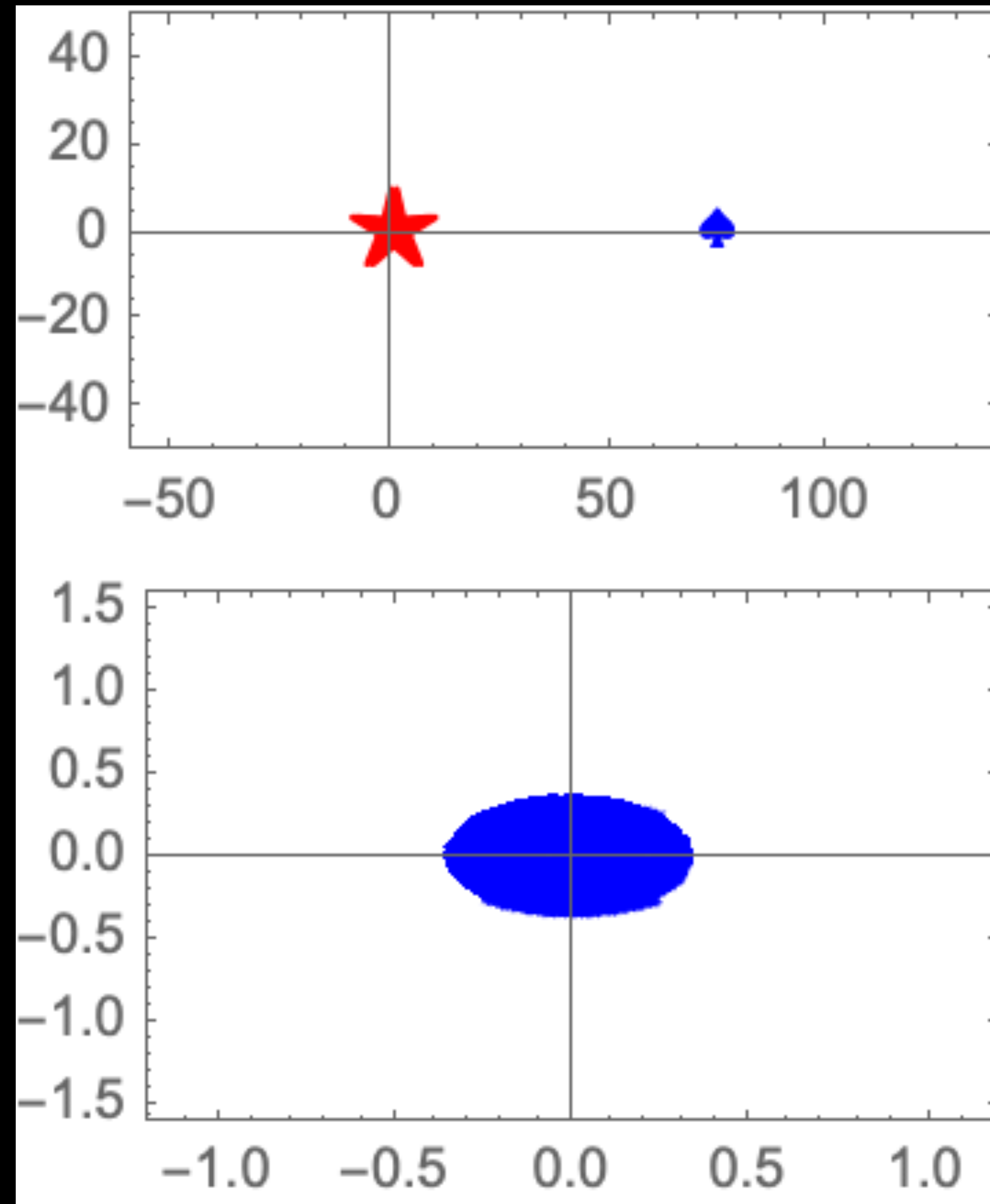
The intensity tends to cluster in the centre-of-mass for the individual lumps.

Interference effects are also there (though a bit hard to see)

Tidal disruption of hydrogen "snow clouds"

- Some non-negligible content of dark matter may be in the form of cold, gas clouds, primarily composed of molecular hydrogen (and some maybe metals scooped up from the Galaxy)

- Sometimes these clouds might get disrupted by stars, ripping away and turning into filaments which can act as lenses. The phase function in this case is essentially traced by the column density.



$$\Psi(\boldsymbol{x}_{obs}, \boldsymbol{x}_s; \nu) \propto \int d\boldsymbol{x}\ \exp\left[i\frac{\omega}{2c}\left(\frac{(\boldsymbol{x}-\boldsymbol{\mu})^2}{\bar{d}} - \frac{\Sigma_e(\boldsymbol{x})e^2}{m_e\epsilon_0\omega^2}\right)\right]$$

Physical optics

$$\phi(\boldsymbol{x}) = \phi^{(0)}(\boldsymbol{x}) - \frac{\omega^2}{\pi}\int d^3\boldsymbol{x}'\,\frac{e^{i\omega|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|}U(\boldsymbol{x}')\phi^{(0)}(\boldsymbol{x}')$$

2D Projection

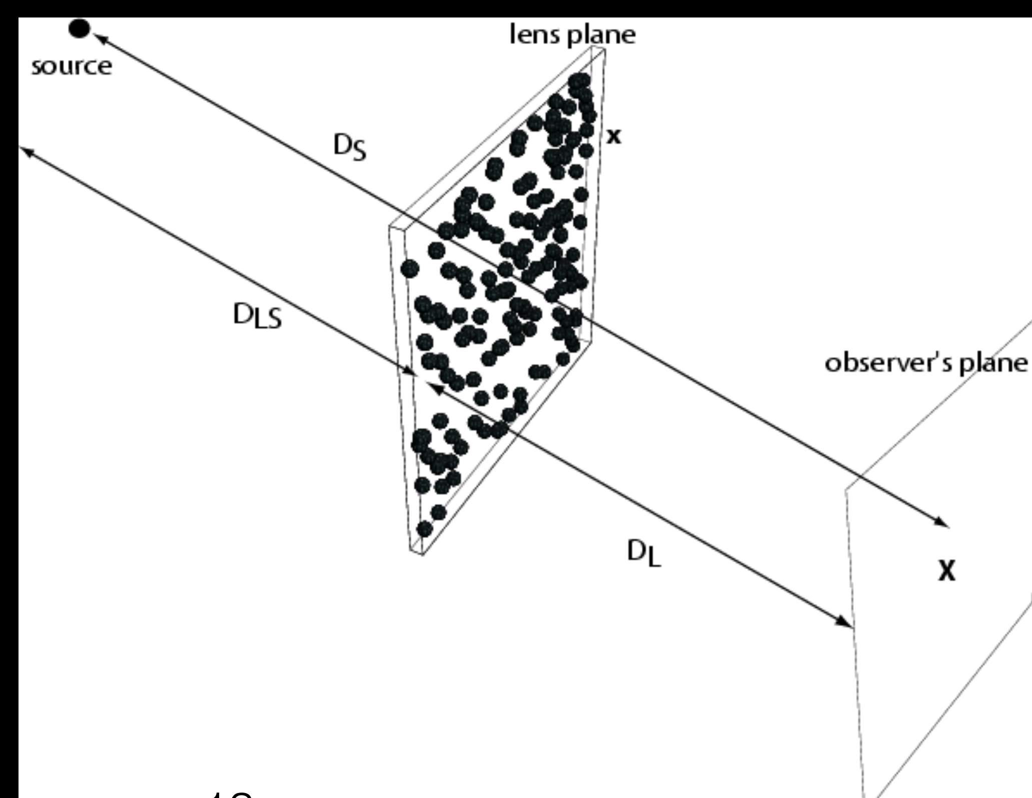
$$F(\boldsymbol{x}_s) = \frac{w}{2\pi i}\int_{\mathbb{R}^2} d^2\boldsymbol{x}\,\exp\left(iw\left[\frac{|\boldsymbol{x}-\boldsymbol{x}_s|^2}{2} - \sum_{j\leq N}\left(\frac{M_j}{M_L}\right)\log\sqrt{(x-x_j)^2+(y-y_j)^2}\right]\right)$$

Pythagorean contribution

Shapiro delay

- We look at here a 2D projection, that it is to assume that the lenses are relatively far away from either the source or the observer; collect them all on a plane and collapse the z dimension.

- Maybe inappropriate, for example, for a binary and the sun (see Marchant++ 2021).



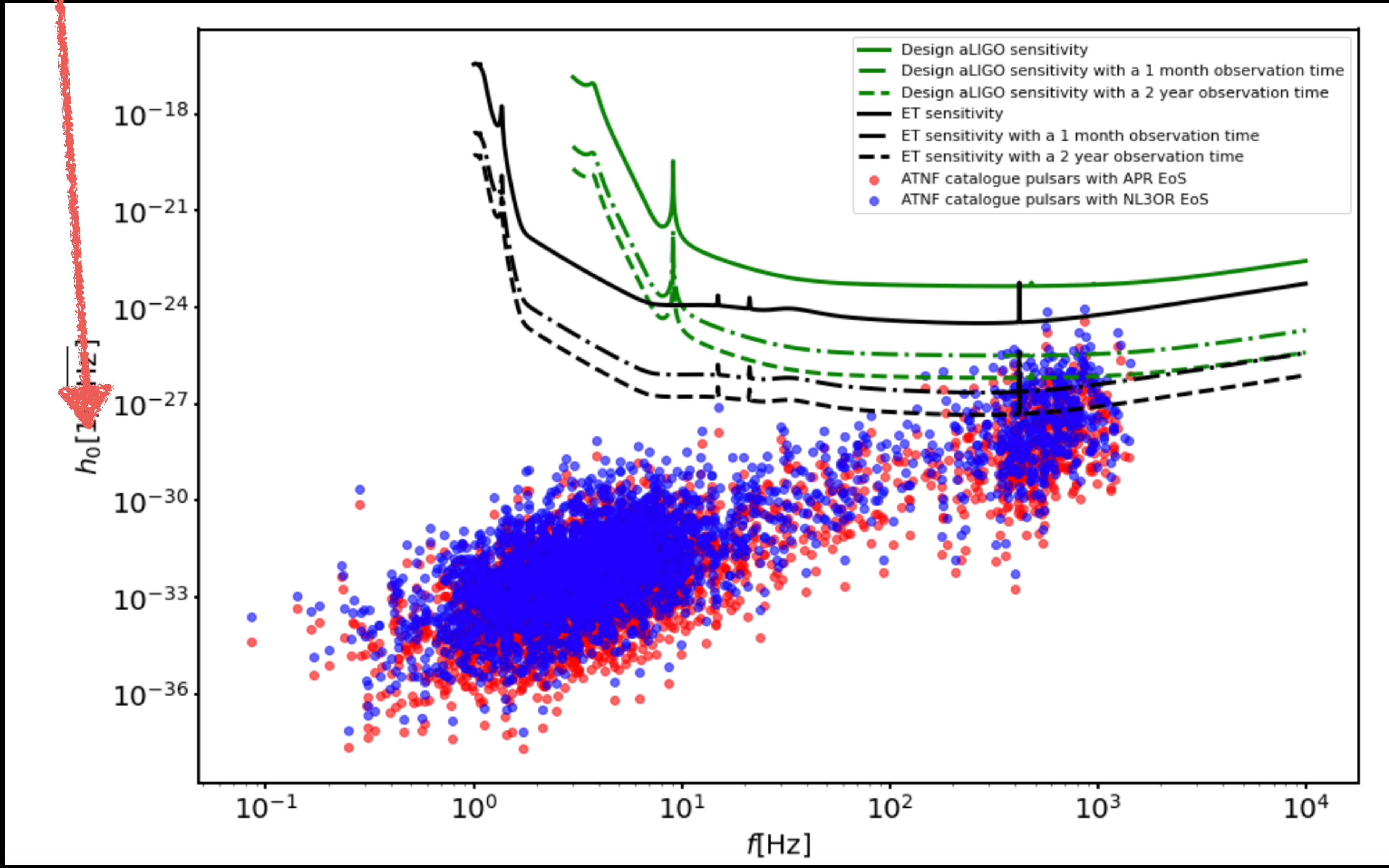
Macquardt (2004), A&A 422, 761

Continuous GWs from neutron stars

- For rapidly rotating neutron stars, several mechanisms can organically (but also inorganically, e.g., accretion) induce large momentum or energy fluxes within the stellar interior.
- It is hoped that these waves will be detected in the near future;
- However, owing to their faintness, it will likely require long periods of persistent monitoring

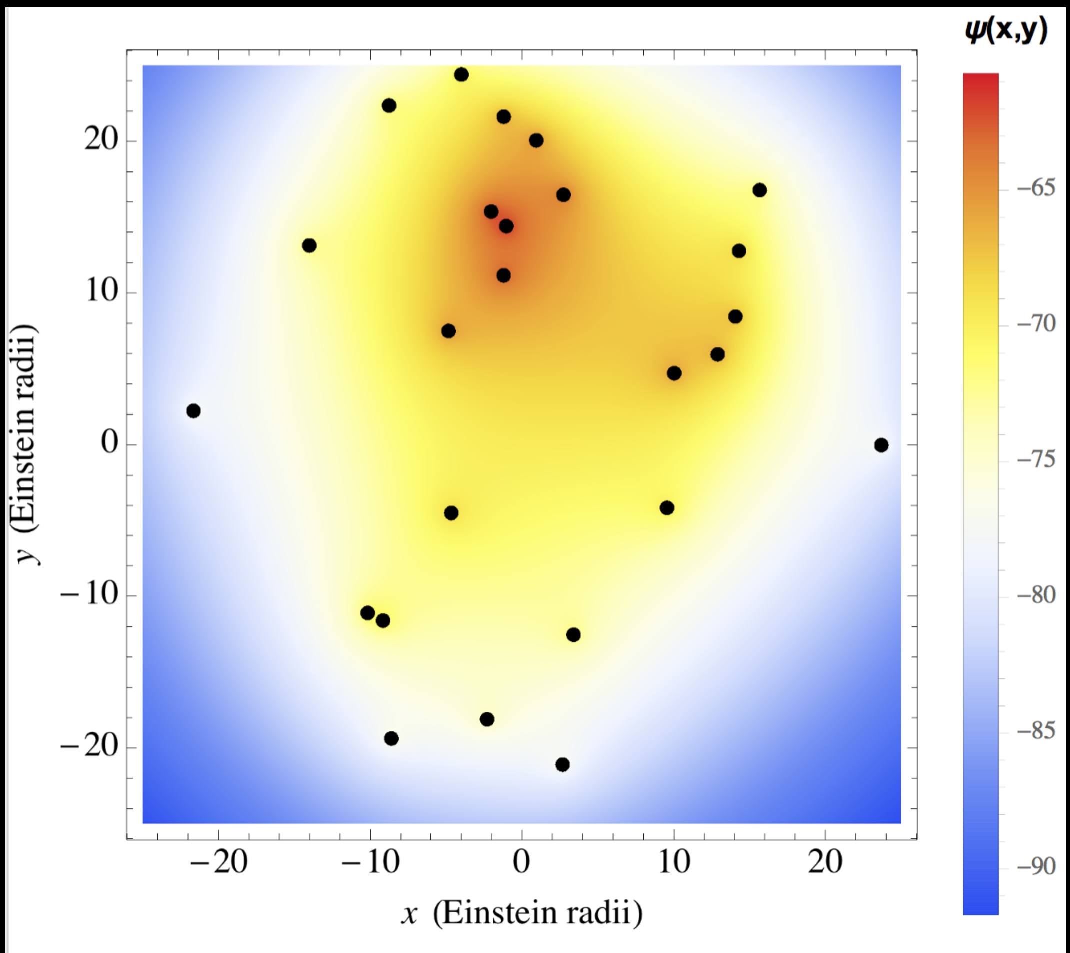
$$h_0 \approx 1.4 \times 10^{-28} \left(\frac{Q_{22}}{10^{36}\ \mathrm{g\ cm}^2}\right) \times \left(\frac{\nu_\star}{500\ \mathrm{Hz}}\right)^2 \left(\frac{10\ \mathrm{kpc}}{D_{OS}}\right),$$

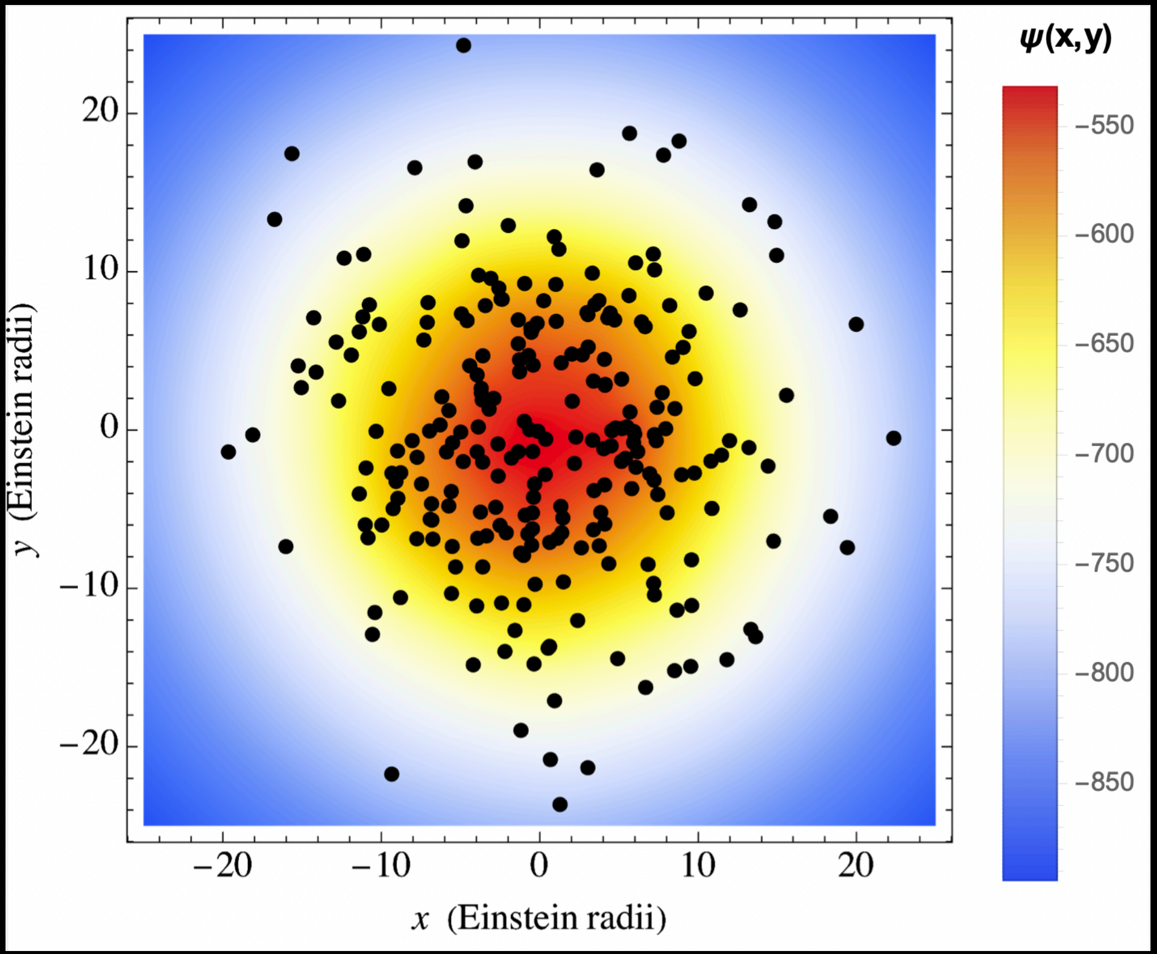
$$Q_{22} \approx 7 \times 10^{35} \left(\frac{B_\star}{10^{13}\ \mathrm{G}}\right)^2 \left(\frac{R_\star}{10^6\ \mathrm{cm}}\right)^6 \times \left(\frac{1.4 M_\odot}{M_\star}\right) \mathrm{g\ cm}^2,$$



Magnetic deformation estimates; Soldateschi & Bucciantini 2021

Lensing in globular clusters





Suppose that the line of sight between the deformed neutron star and Earth intersects with a star cluster.

For example, the globular cluster Tuc 47 lies ~4 kpc from Earth, contains ~10^5 stars and at least 27 millisecond pulsars. (Freire++2017; Ridolfi++2021)

When is it really wave optics?

$$\Omega \approx 1.2 \times 10^5 (M/M_\odot)(\nu/\mathrm{GHz})$$

Geometric optics valid for Omega >>1

- In this talk we are interested in the long wavelength limit.

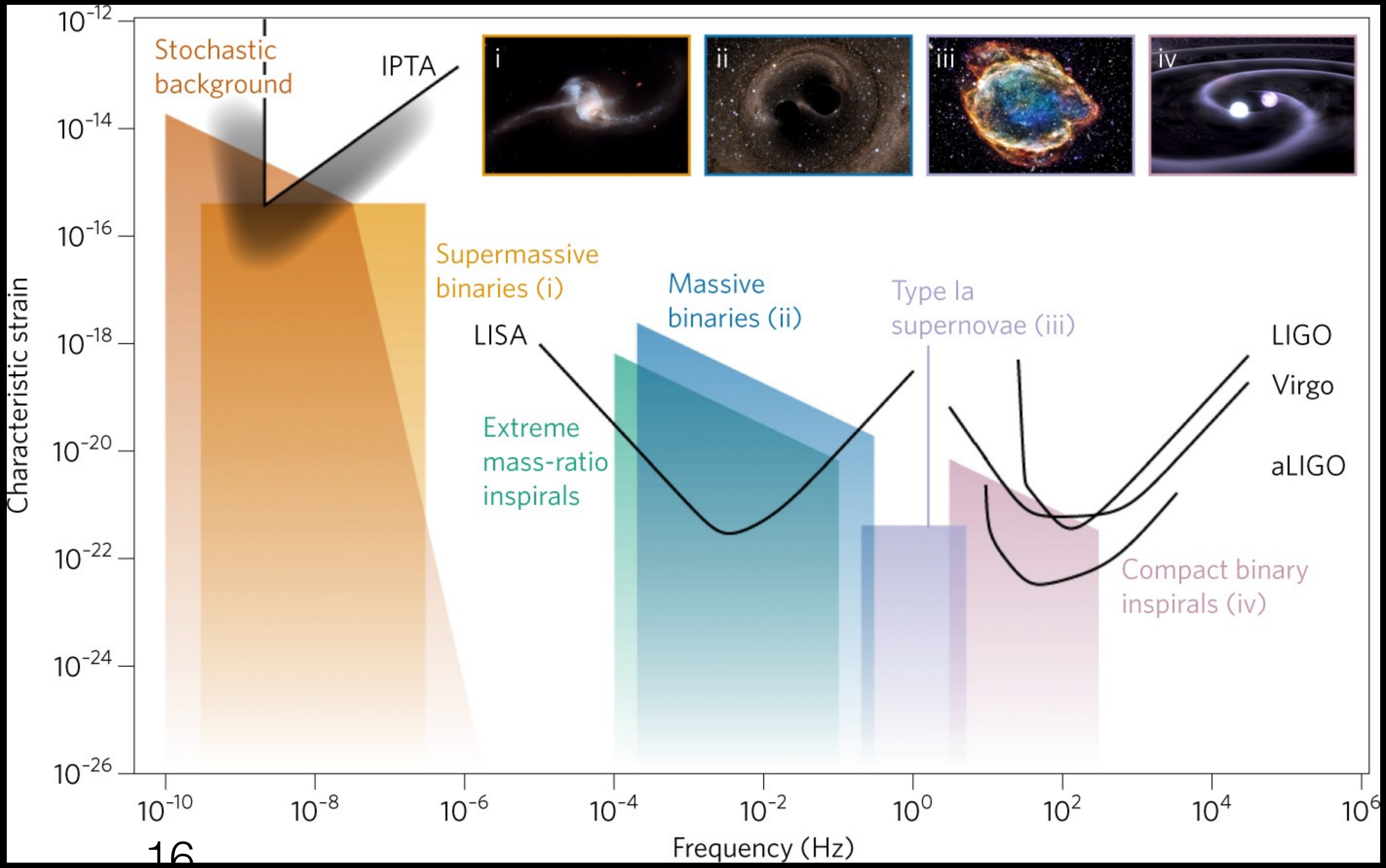
- For EM radiation this is likely uninteresting for gamma-rays, and so on. But for radio there can be planetary mass lenses, which has all sorts of interesting implications as concerns radio pulsars and FRBs.

- Here: radio emissions and GWs!

Diffraction for:

EM: ~planets

GW: ~stars



GWPlotter; Moustakas 2017

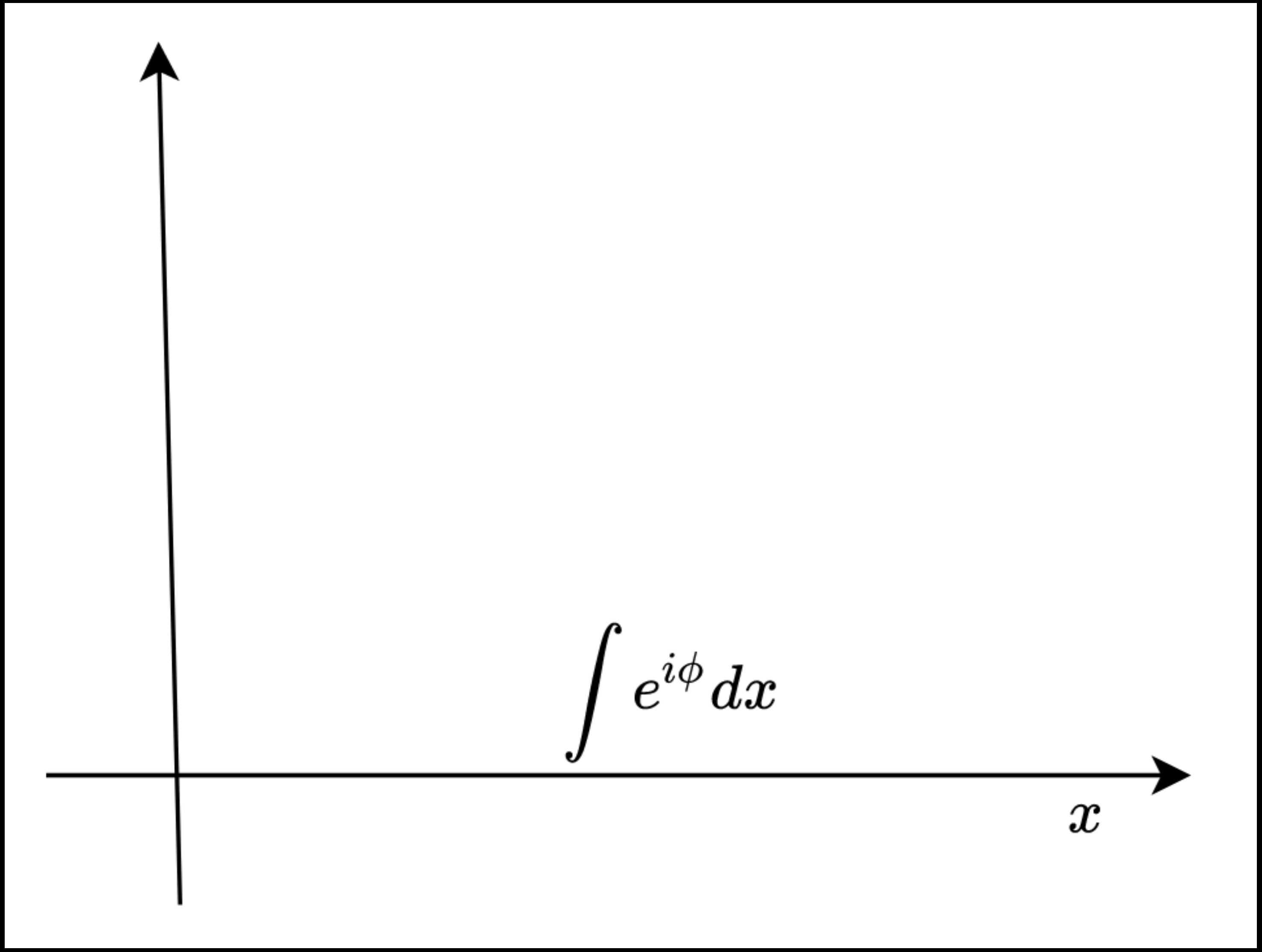
Promotion to the complex plane

- Eventually, we want to evaluate integrals that look like this:

$$\int e^{i\phi}\,dx \sim \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\,e^{i\nu t_d(x,y,x_s,y_s)}$$

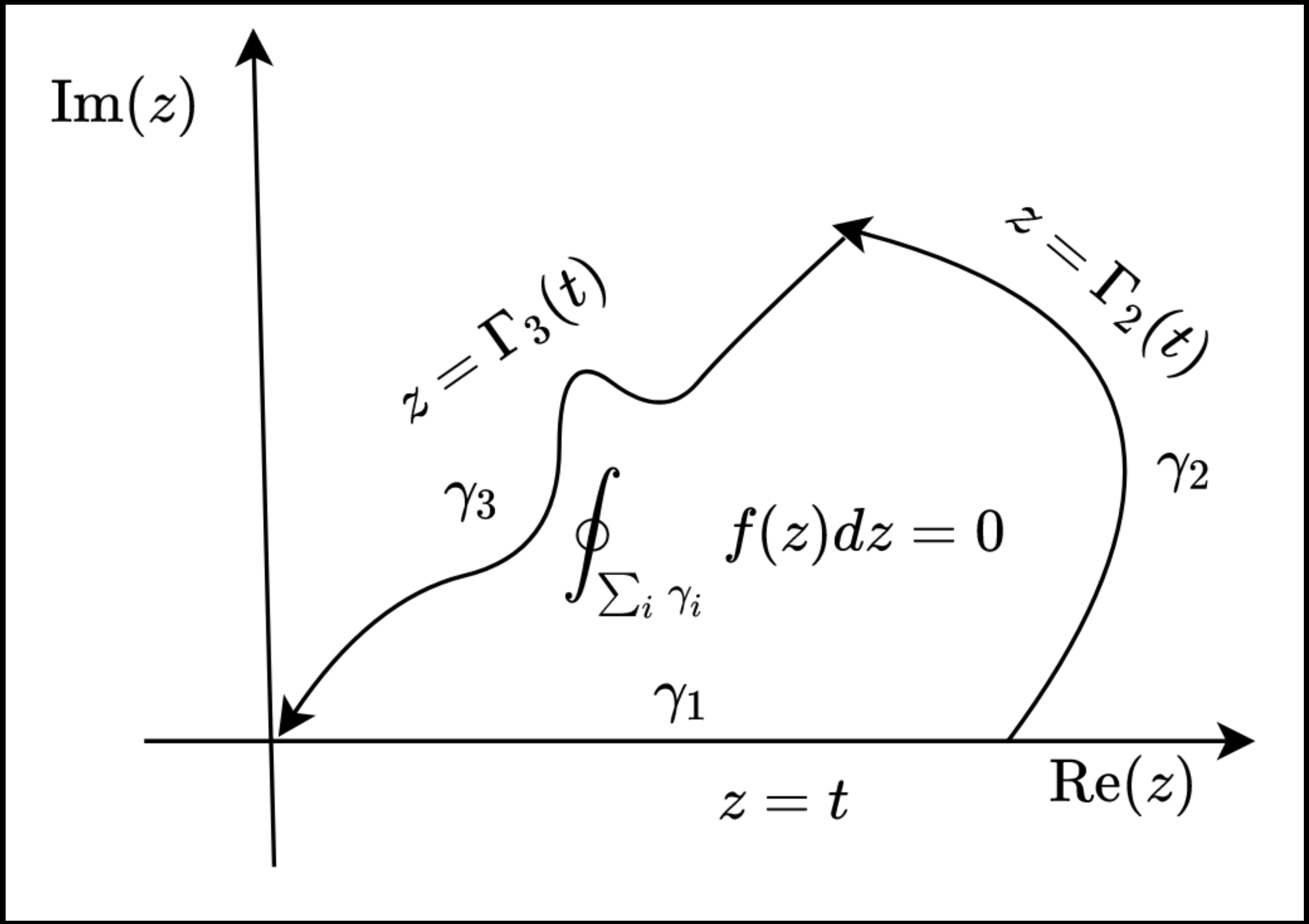
- The oscillatory integral can be thought of a line integral over the reals (trivially)

- Cauchy's theorem can then be used by deforming the "contour" into the complex plane



Promotion to the complex plane

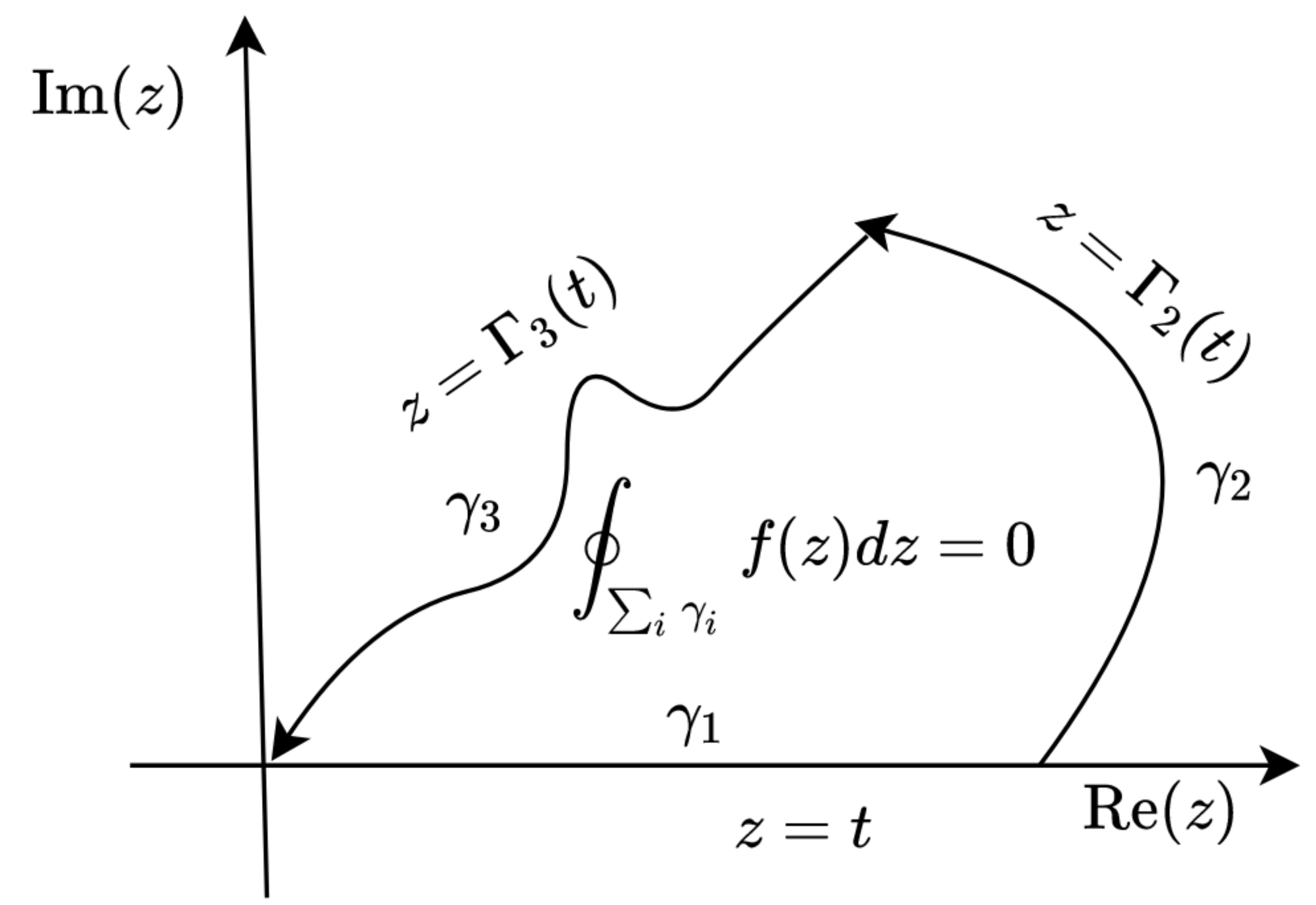
- Cauchy's theorem tells us that the integral, provided the function is holomorphic (complex analytic), is zero



Promotion to the complex plane

- Cauchy's theorem tells us that the integral, provided the function is holomorphic (complex analytic), is zero

- Since the sum is zero, we can evaluate the real integral without actually evaluating it, but evaluating two other integrals instead



Cauchy

$$\Longrightarrow$$

$$\int e^{i\phi} dx = -\int_{\gamma_2} e^{i\phi[\Gamma_2(t)]}\dot{\Gamma}_2(t)dt - \int_{\gamma_3} e^{i\phi[\Gamma_3(t)]}\dot{\Gamma}_3(t)dt$$

Tends to vanish by Jordan's lemma