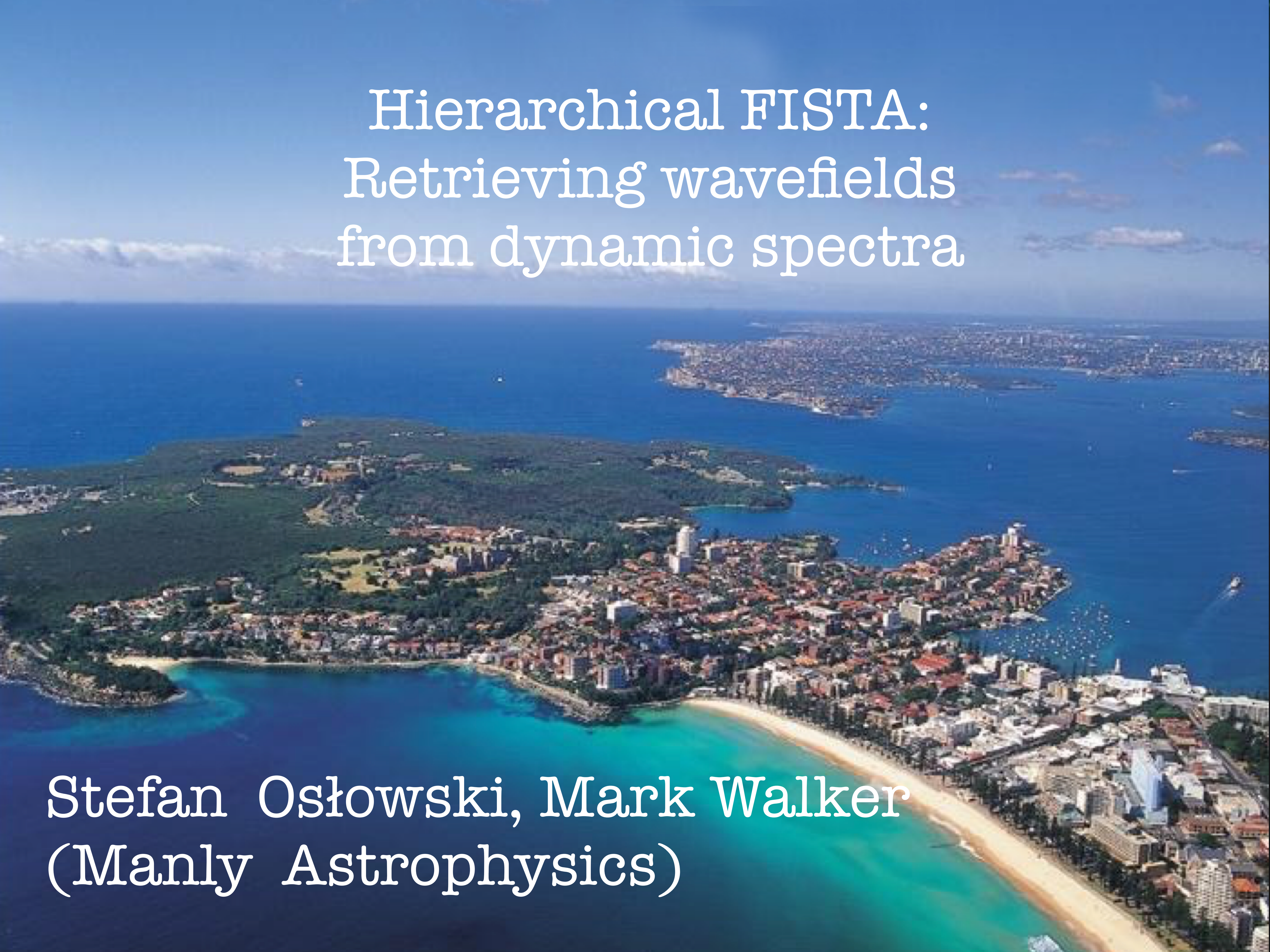


Hierarchical FISTA: Retrieving wavefields from dynamic spectra

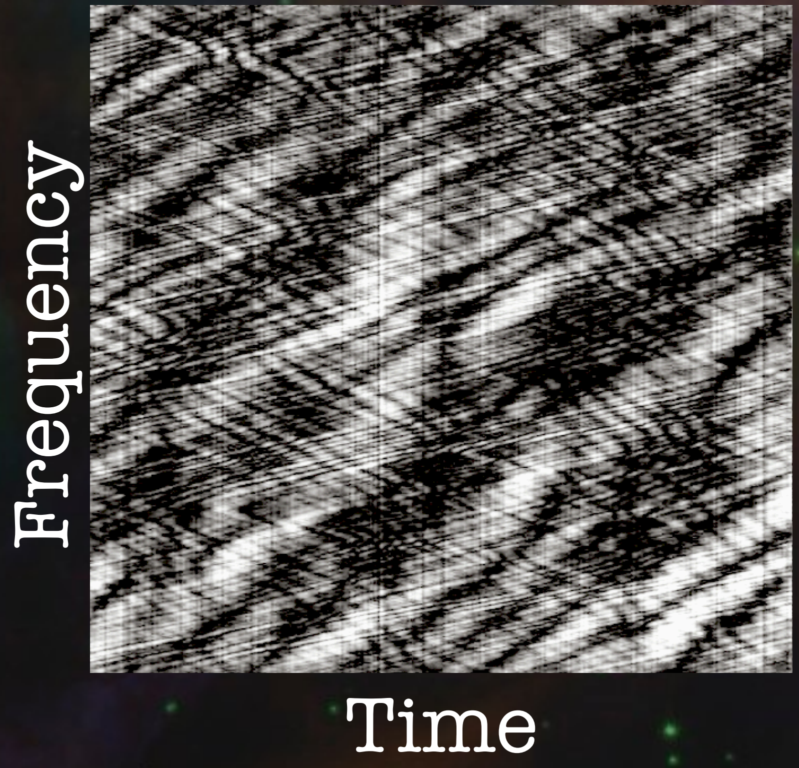
Stefan Osłowski, Mark Walker
(Manly Astrophysics)

Fourier Relationships

Dynamic Spectrum

2nd order in field

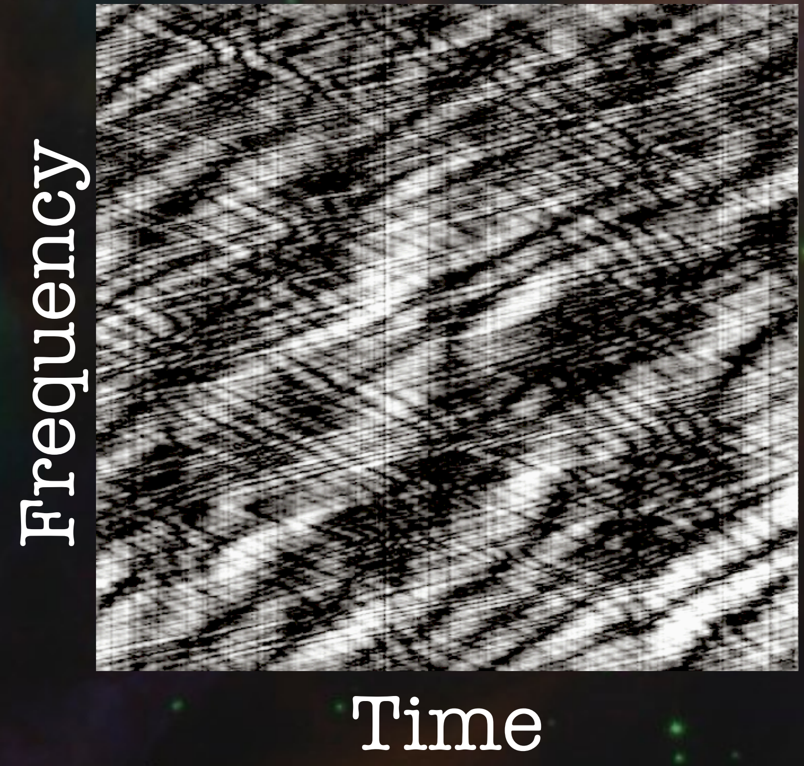
$$I(\nu,t)=|H(\nu,t)|^2$$



Fourier Relationships

Wavefield	Dynamic Spectrum	"Secondary Spectrum"
1^{st} order in field	2^{nd} order in field	4^{th} order in field

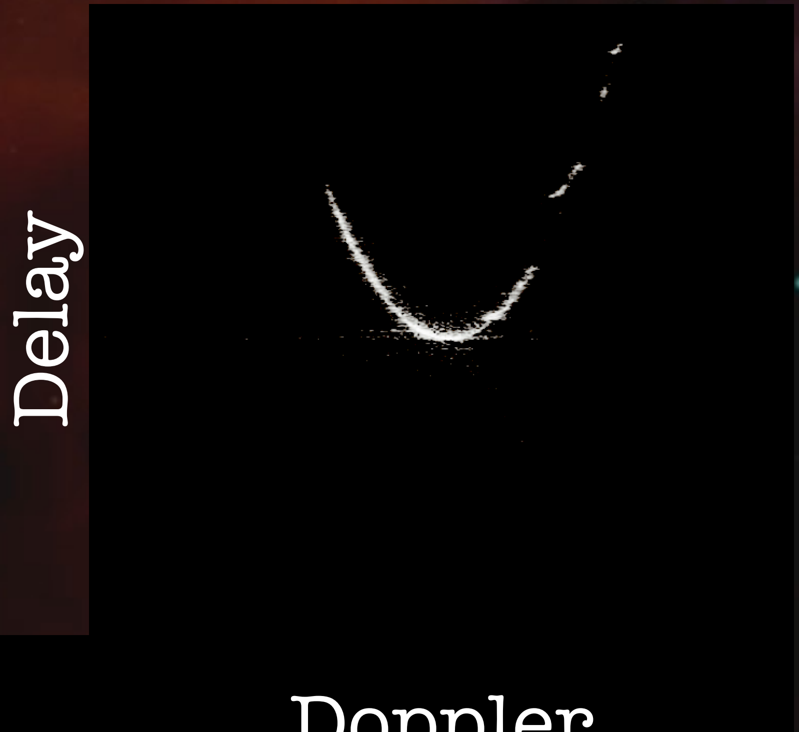
$$I(\nu,t)=|H(\nu,t)|^2$$



Power Spectrum

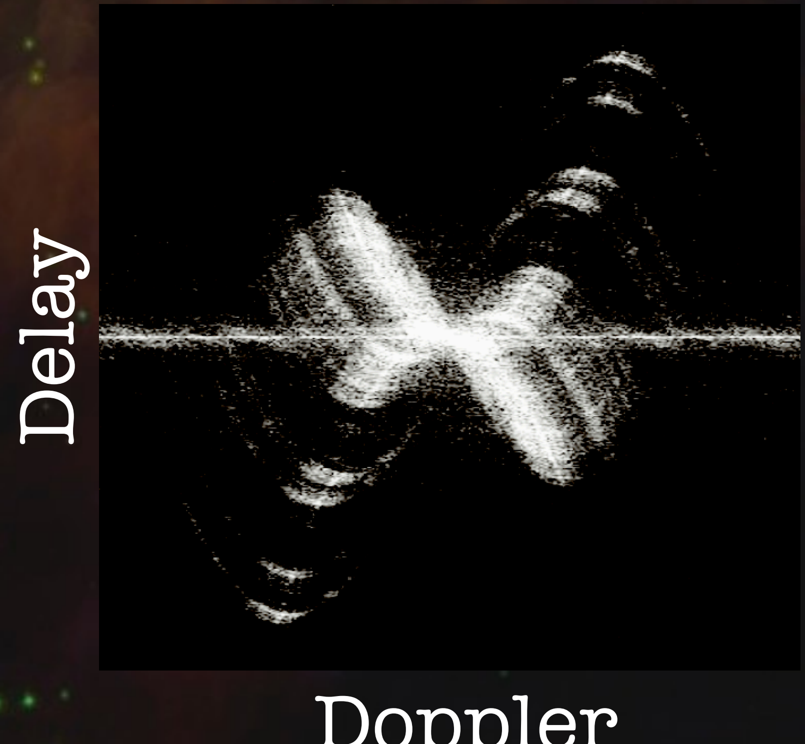
Power Spectrum

$$h(\tau,\omega)$$



$$i(\tau,\omega) = h \otimes h^*$$

$$s(\tau,\omega)=|i(\tau,\omega)|^2$$



Obtaining a sparse wavefield

- Use LASSO / 1-norm regularisation
- Minimize $F(h) = f(h) + g(h) = R(h)^2 + \lambda|h|$

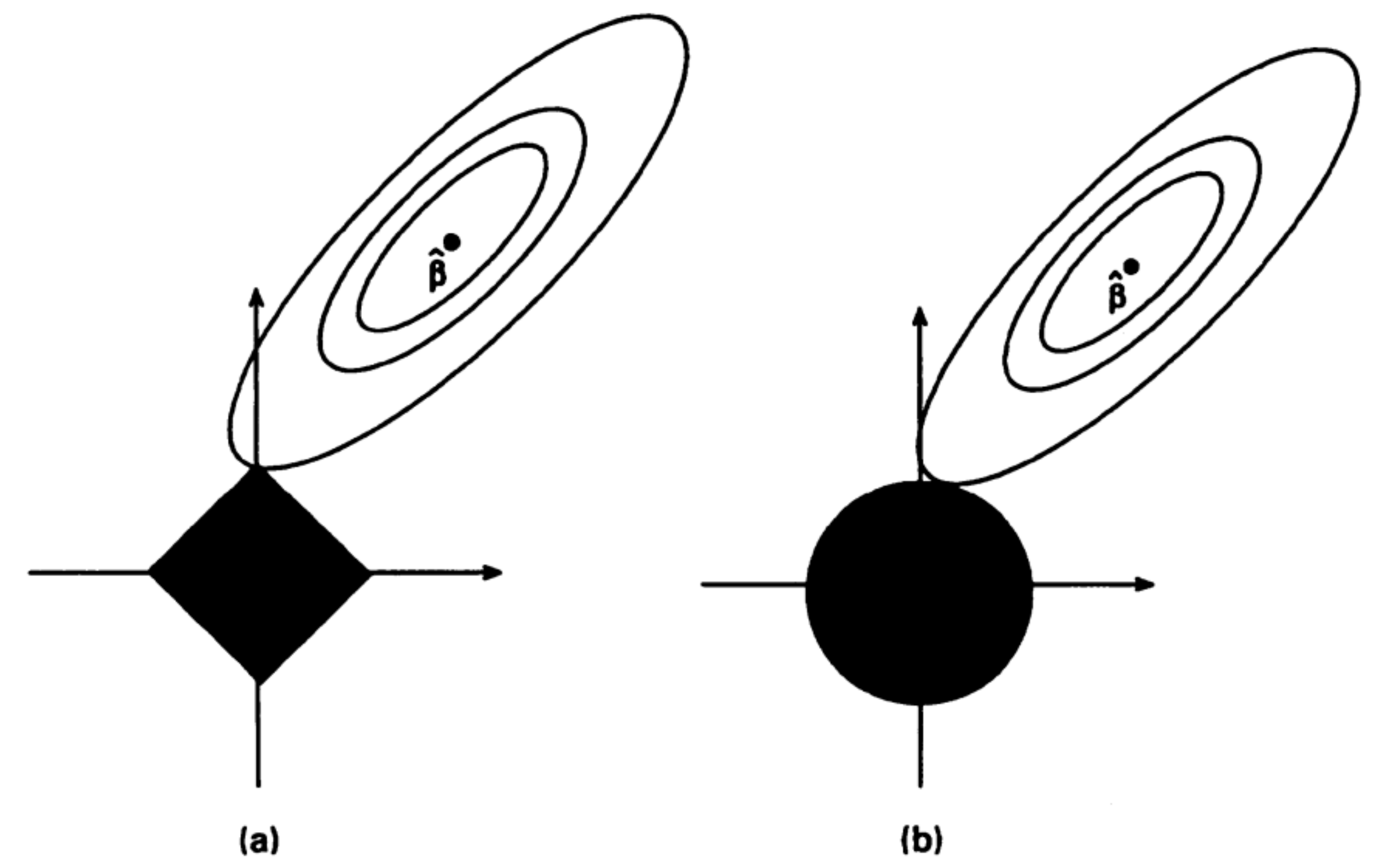


Fig. 2. Estimation picture for (a) the lasso and (b) ridge regression

Tibshirani, 1996

Obtaining a sparse wavefield

- Use LASSO / 1-norm regularisation
- Minimize $F(h) = f(h) + g(h) = R(h)^2 + \lambda|h|$
- Use FISTA - Fast Iterative Shrinkage Threshold Algorithm
- requires knowledge/calculation of the Lipschitz's constant for the gradient

Fast Iterative Shrinkage Threshold Algorithm

Input: L - the Lipschitz constant
λ - regularisation parameter
h_0 - initial model of the wavefield

Step 0: $t_0 = 1$
$y_0 = h_0$

Step k+1: $h_{k+1} = \text{prox}\left(y_k - \frac{1}{L}\nabla f(y_k)\right)$

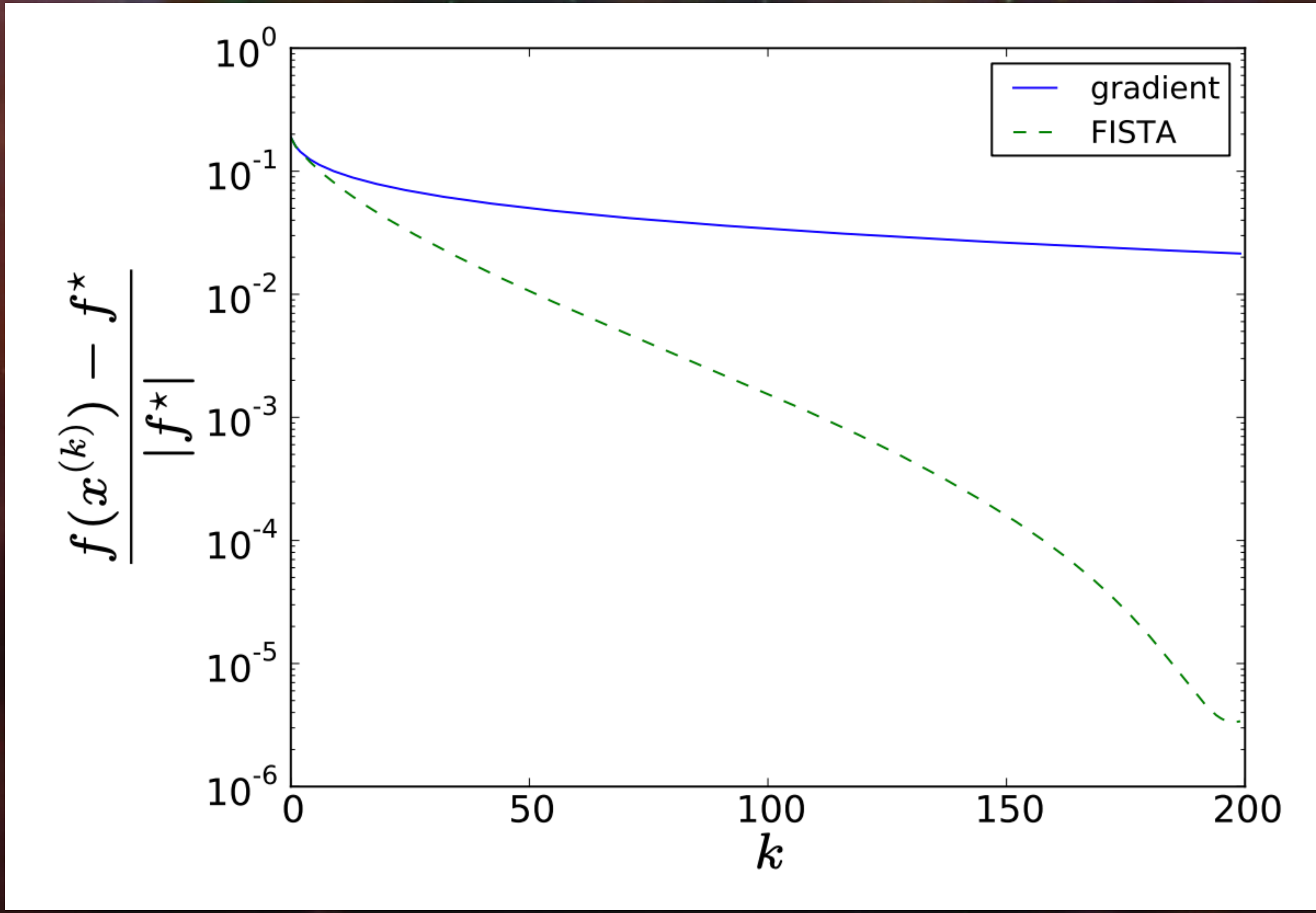
$$t_{k+1} = \frac{1+\sqrt{1+4\,t_k^2}}{2}$$

$$y_{k+1} = h_{k+1} + \frac{(-1+t_k)}{t_{k+1}}\,(h_{k+1} - h_k)$$

$$||\nabla f\,(y) - \nabla f\,(x)\,|| \;\leq\; L\,||y - x||$$

Beck and Teboulle 2009

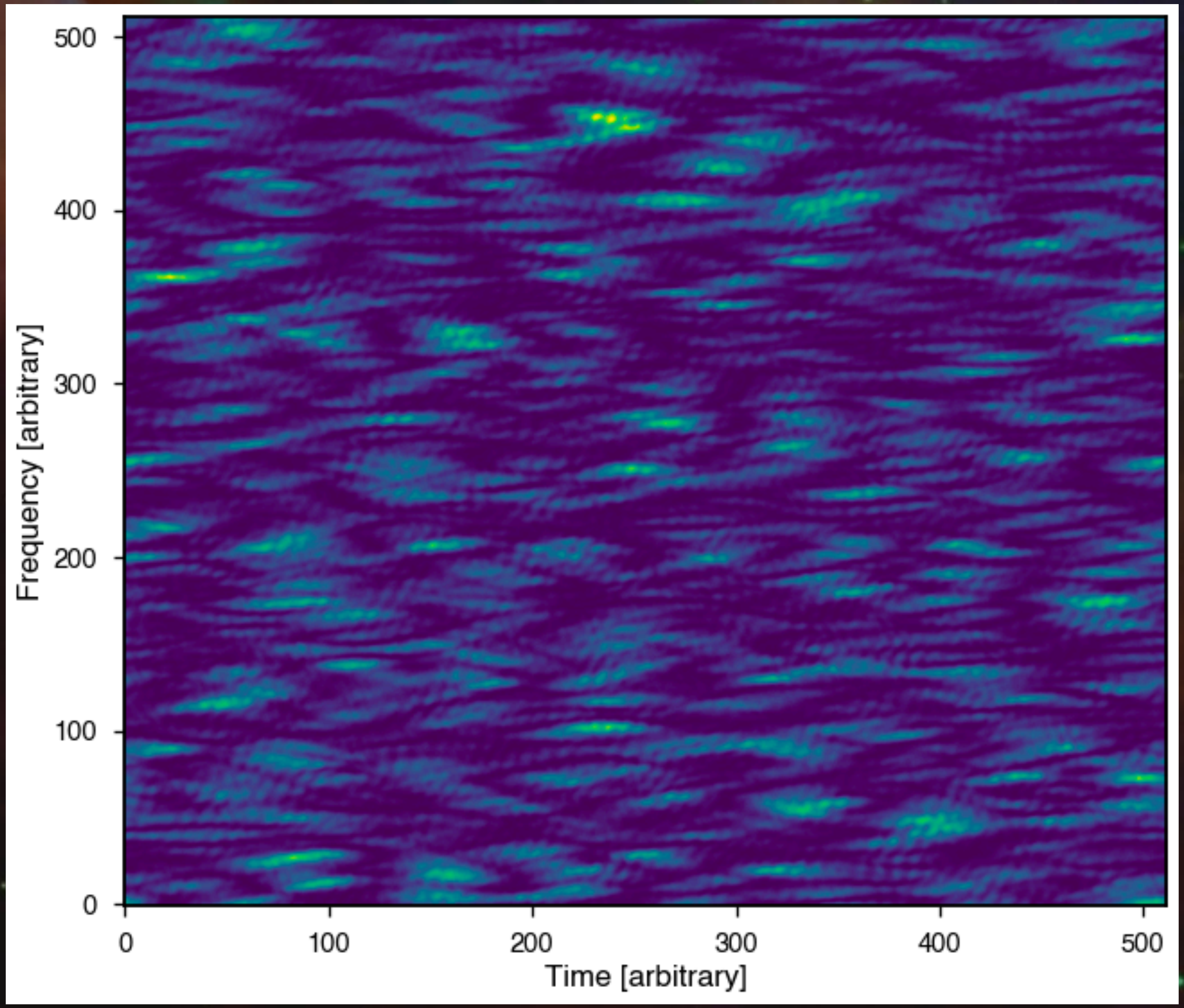
- Fast Iterative Shrinkage Threshold Algorithm



Beck and Teboulle 2009, Vandenberghe 2013

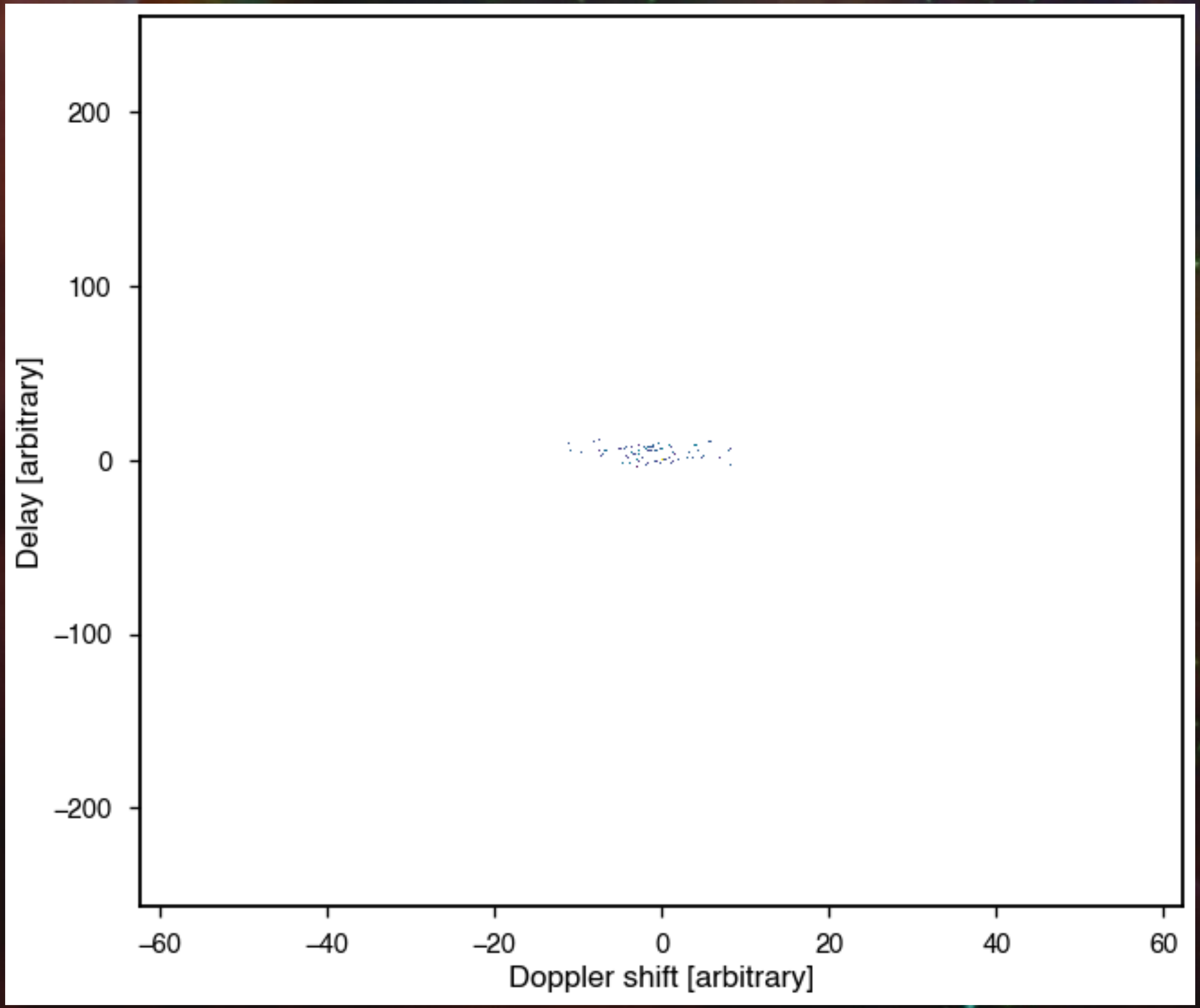
Some details - initial guess

- Initial guess? constant dynamic spectrum
- Run FISTA with $\lambda=\lambda_{init}$



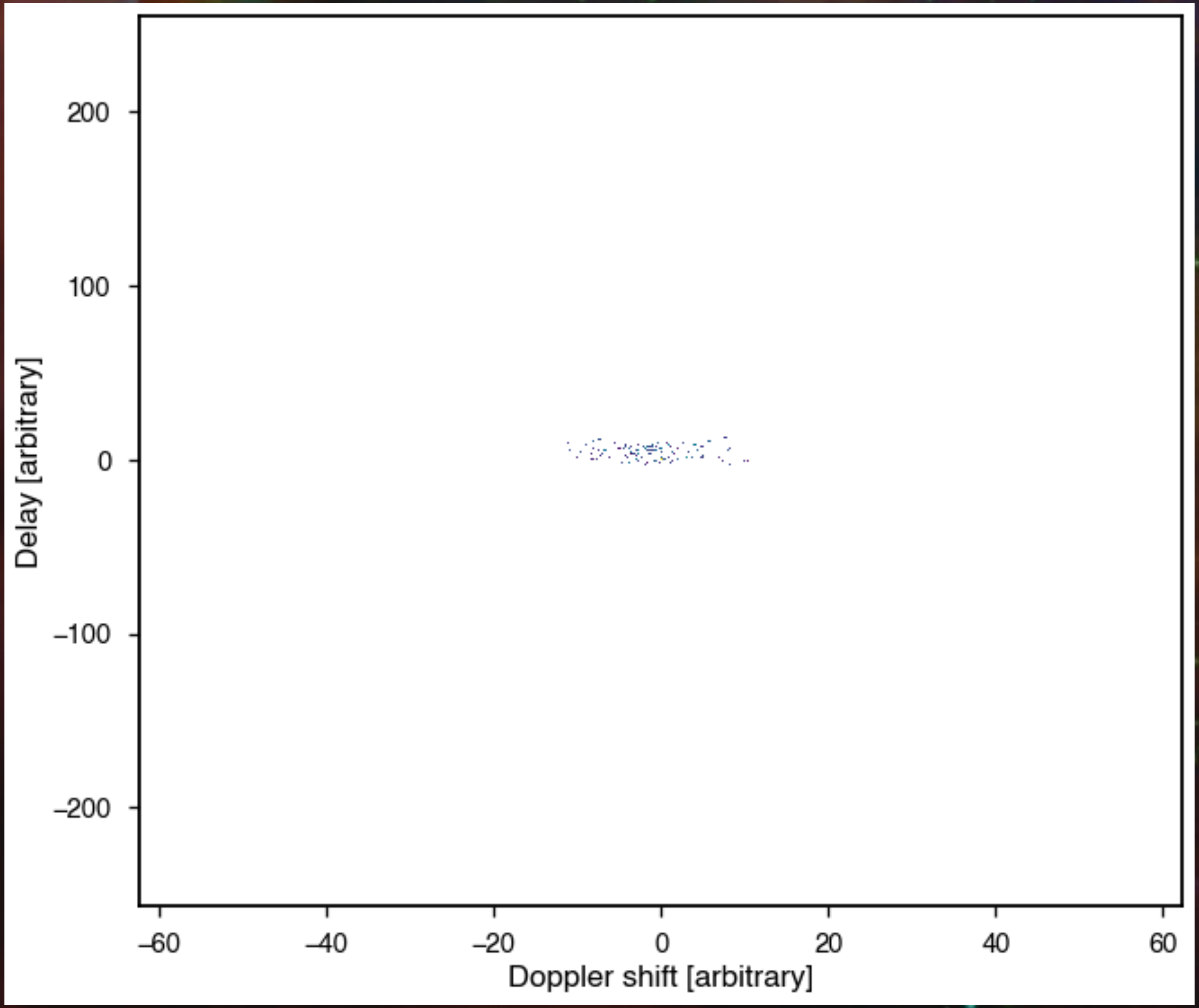
Some details - debias the model

- Apply λ=0 and λ=∞ after FISTA
- Hard-threshold small components



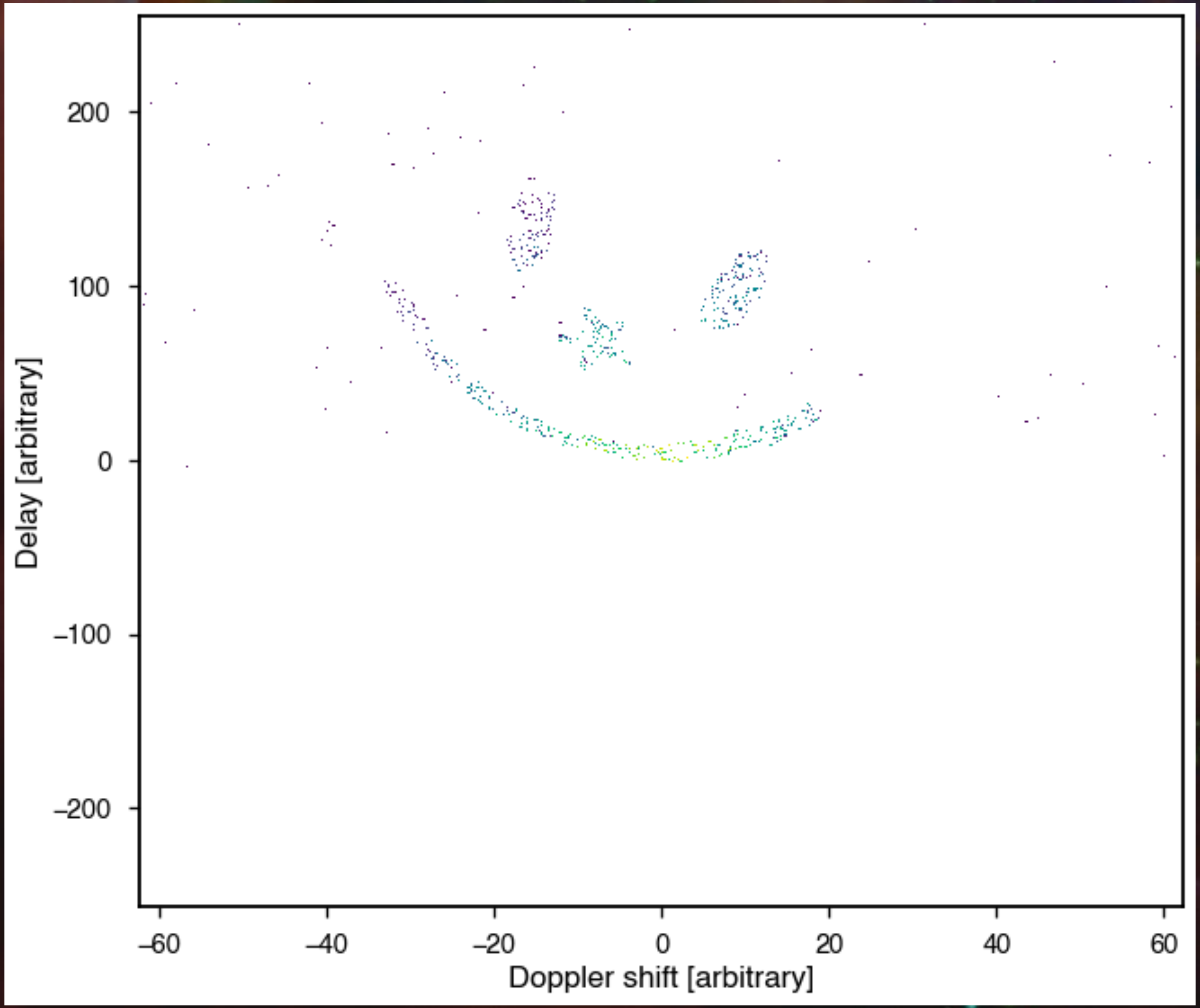
Some details - extending the model

- Run FISTA again with $\lambda=0$ and $\lambda=\lambda_2=\lambda_1/\eta$
- $\eta=1.15$



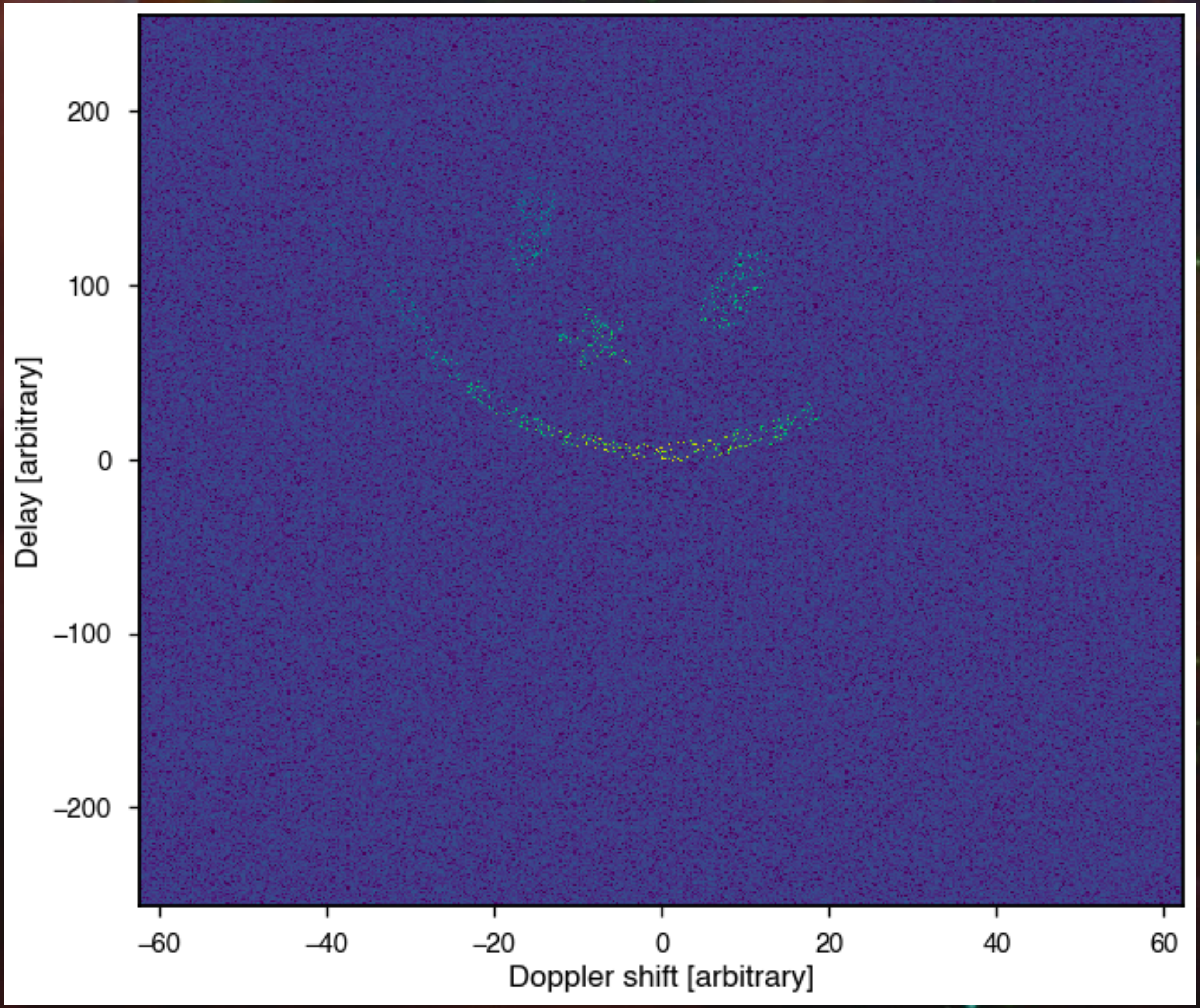
Some details - extending the model

- Repeat till satisfied



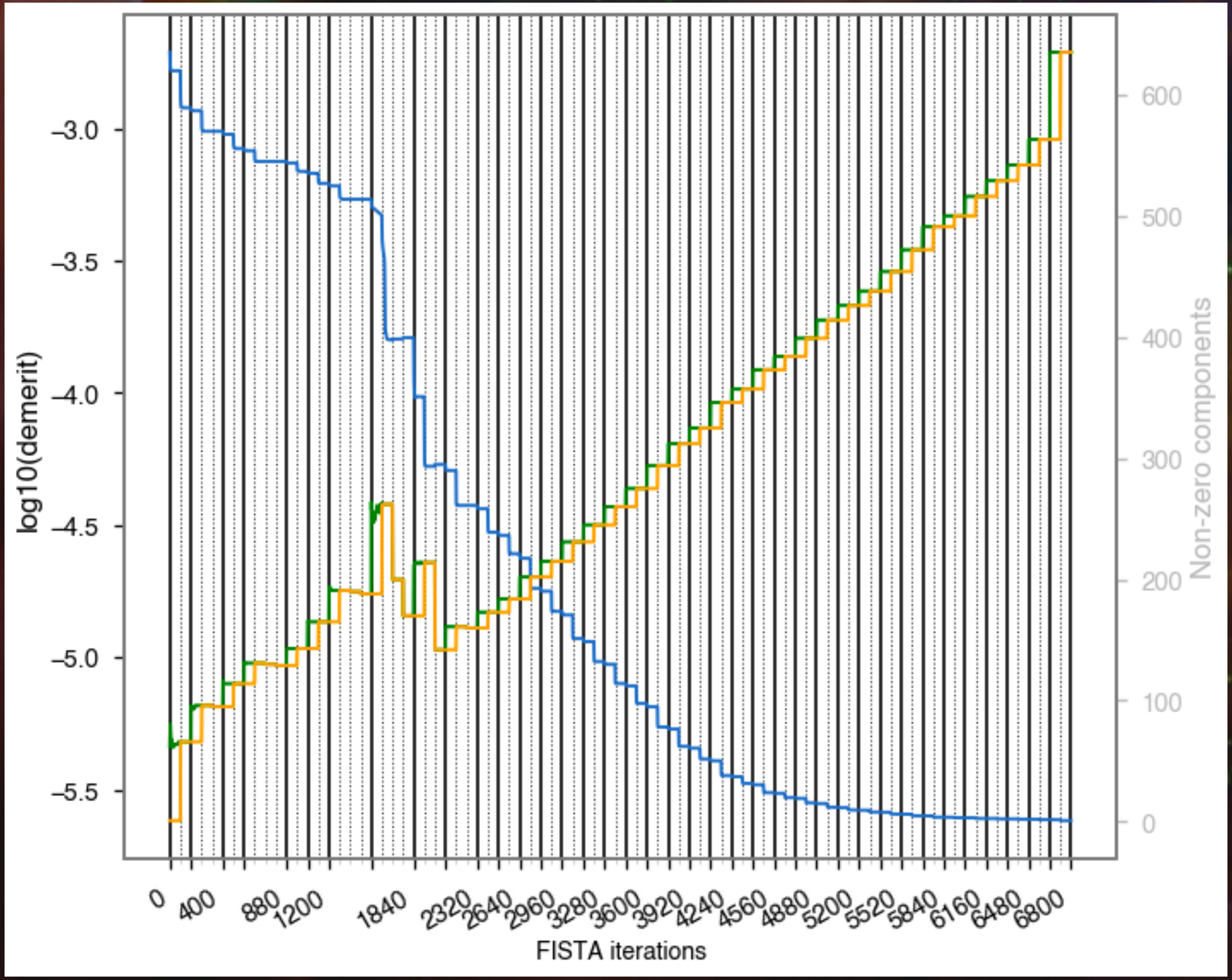
Calculate a dense wavefield

- Unregularised FISTA (or ridge regression)



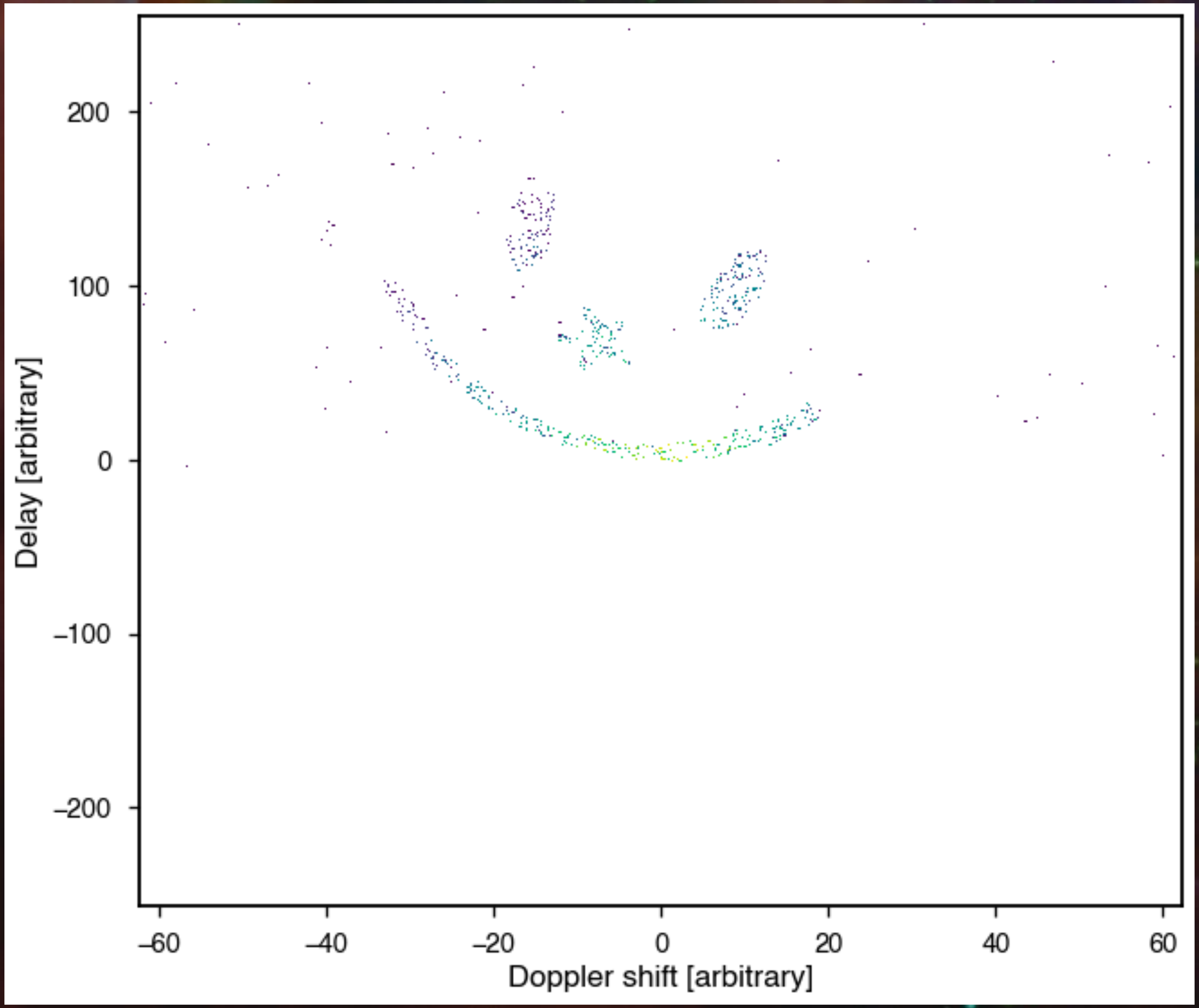
H-FISTA progress

- Demerit and number of components



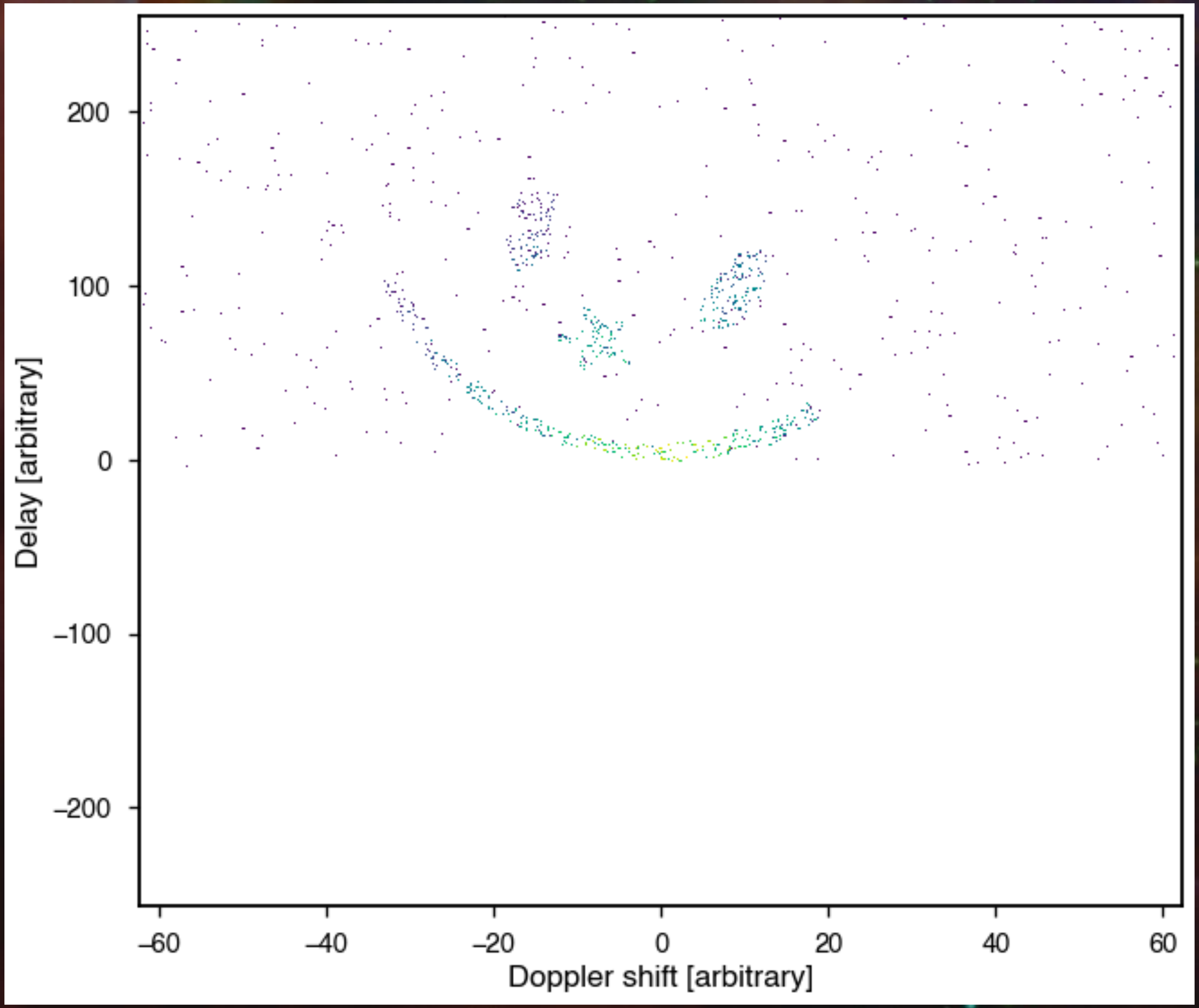
Some details - stopping

- Repeat till satisfied



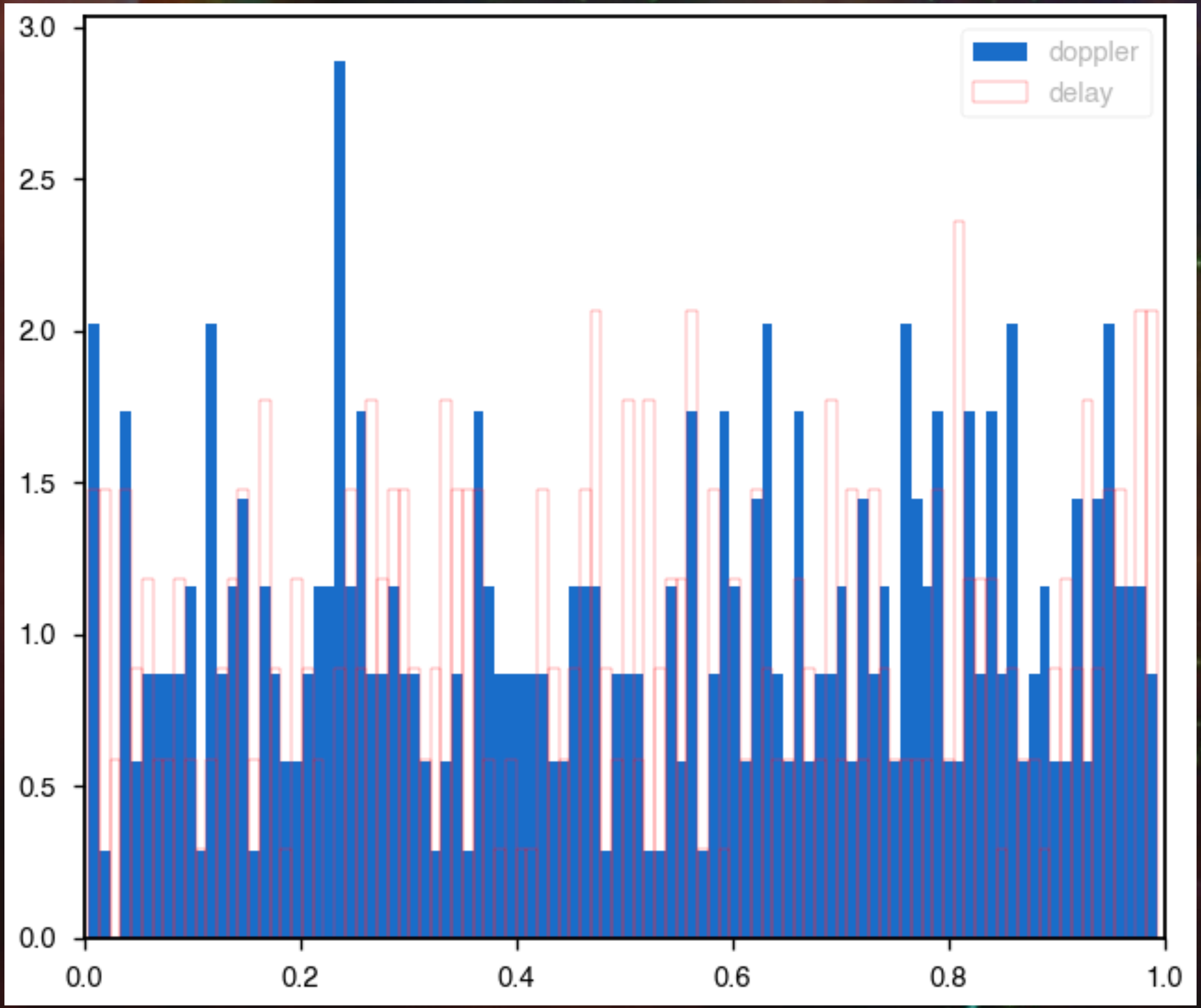
Some details - stopping

- Repeat till satisfied



Some details - stopping

- Check distribution of new components



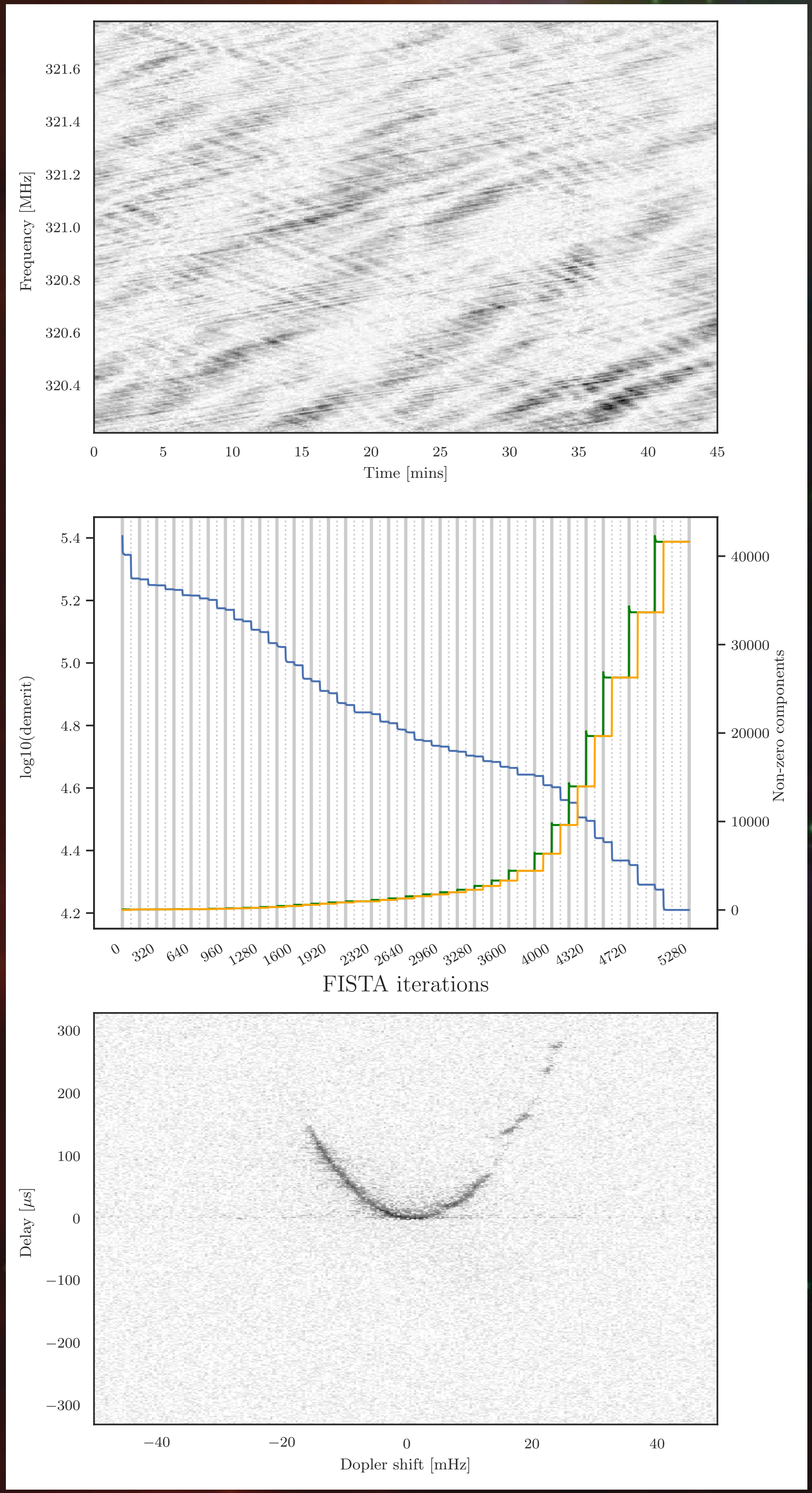
Hierarchical FISTA

Input:	λ_{init} - initial regularisation parameter
	or instead
	N_0 - model components after the very first FISTA step
	ϵ - scale factor for hard thresholding operator T_{H}
	N_{iter} - number of iterations in each FISTA optimisation
	η_λ - λ scaling factor
Step k:	
	1. Set $\lambda_k = \lambda_{k-1}/\eta_\lambda$
	2. Set $\Lambda = 0$ where $h \neq 0$, λ_k elsewhere
	3. Run FISTA
	4. Set $\Lambda = 0$ where $h \neq 0$, ∞ elsewhere
	5. Run FISTA
	6. Perform hard thresholding with threshold $\epsilon\lambda/L$
	7. If any components zeroed, go back to step 4
	8. Check stopping criteria: exit or return to step 1

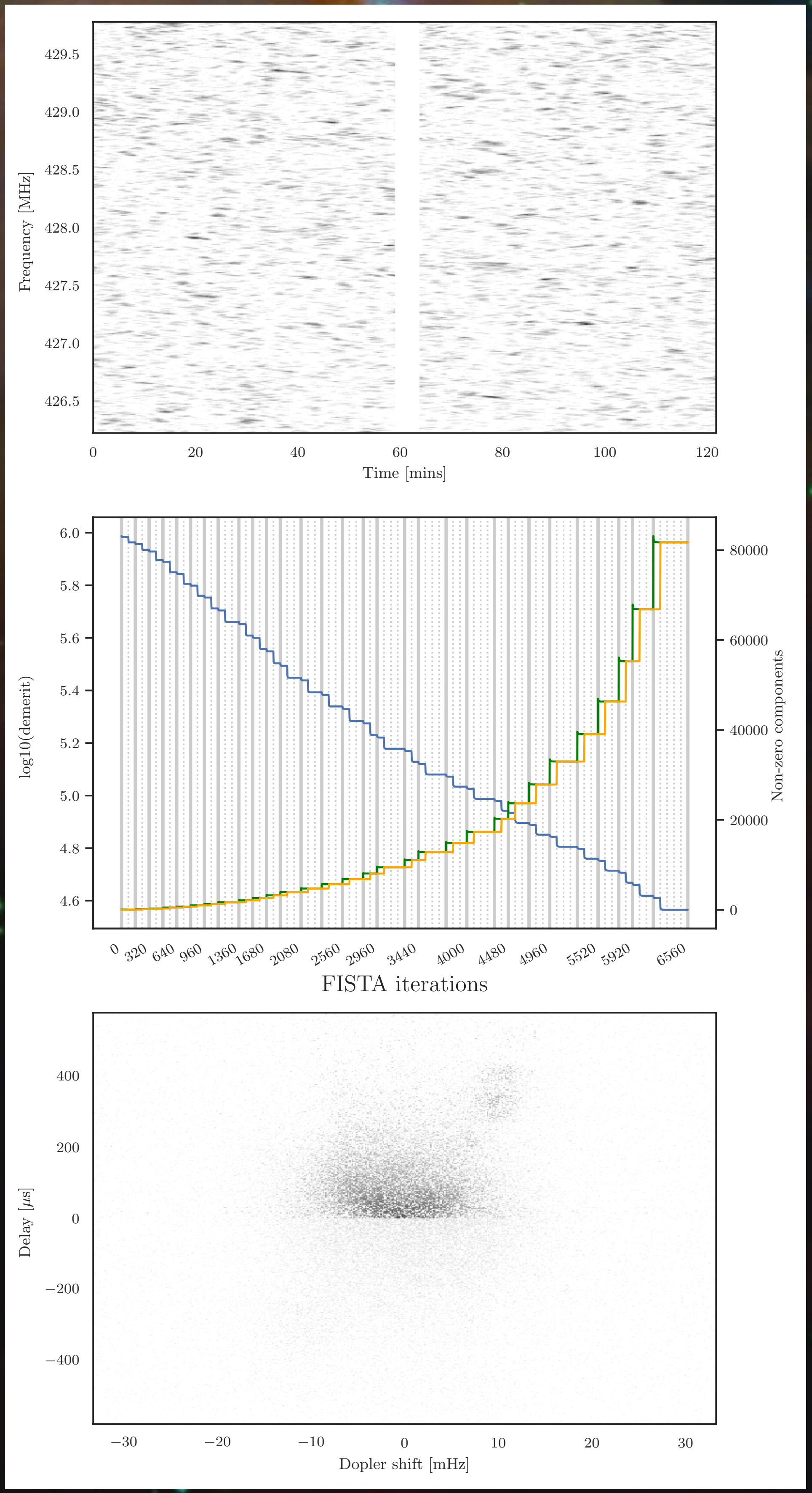
Osłowski & Walker, submitted

Results

J0837+0610



J1939+2134



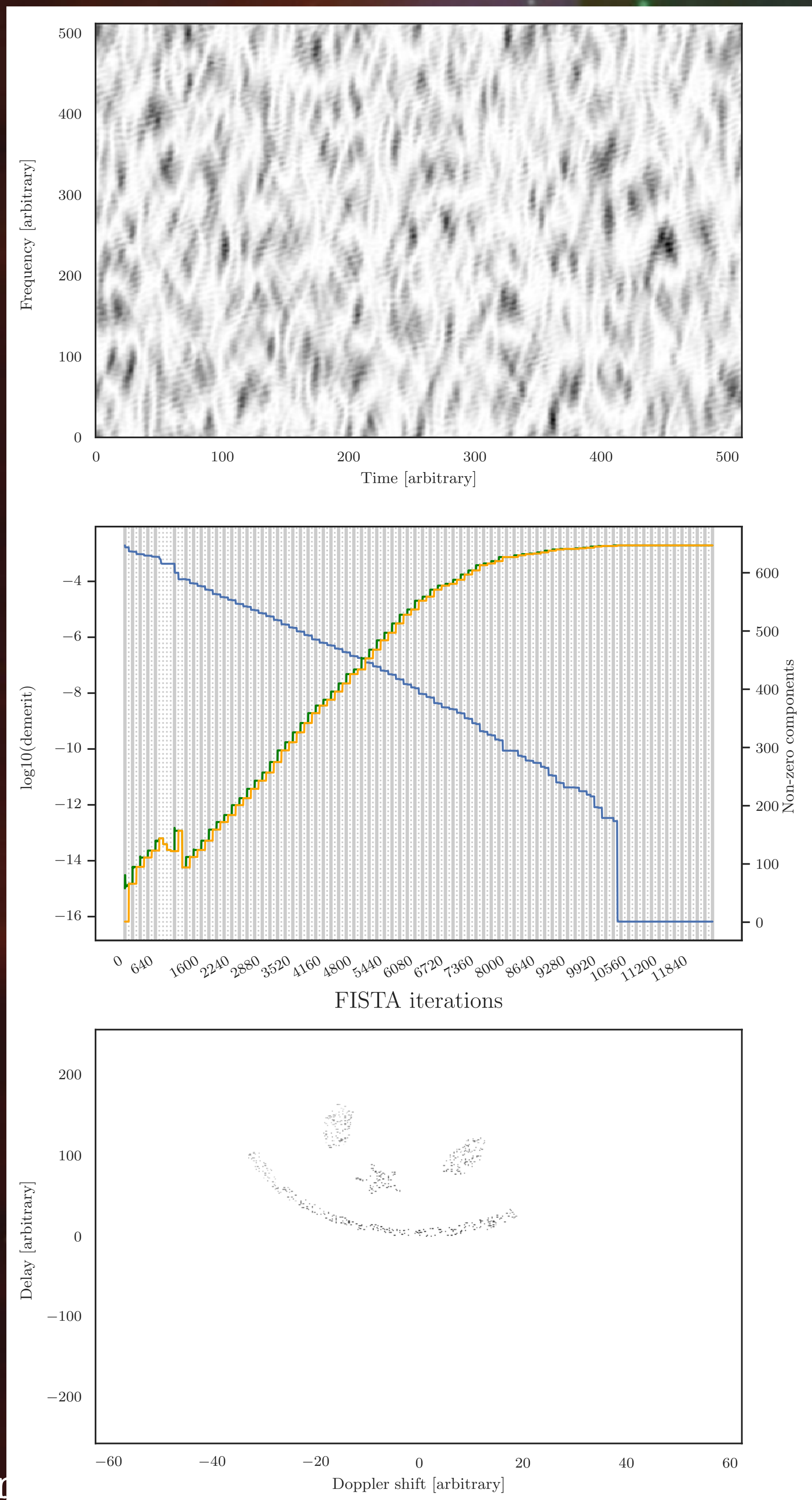
Manly Astrophysics

Summary

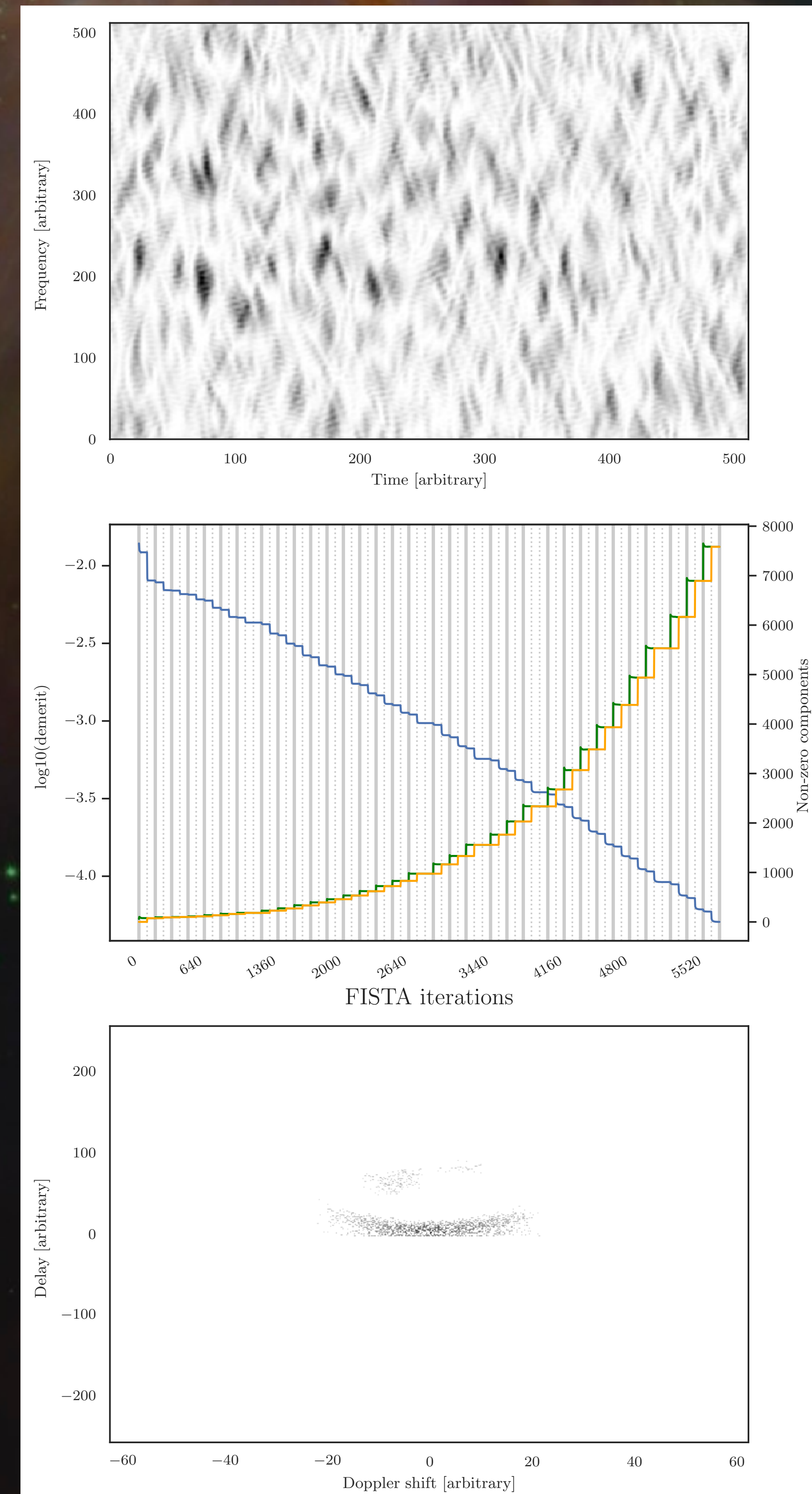
- Successful phase retrieval on simulated and observed dynamic spectra
- Use a rapid optimisation algorithm with progressively reduced regularisation
- Works well on relatively low local density wavefields
- Can struggle to remove twin image from locally dense wavefields


Manly Astrophysics

Simulated noise free spectrum local density 12.5%



Simulated noise free spectrum local density 25%

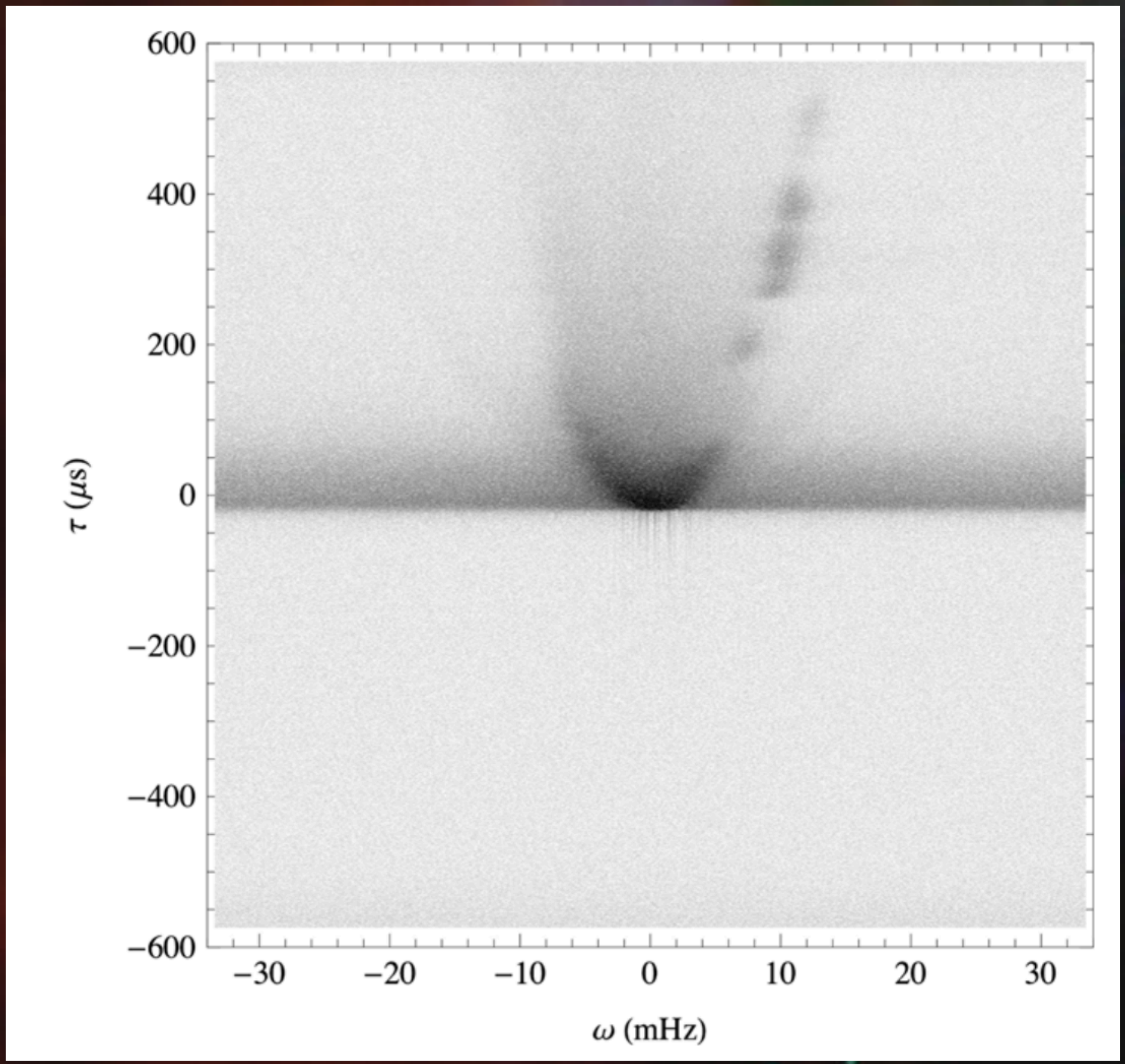


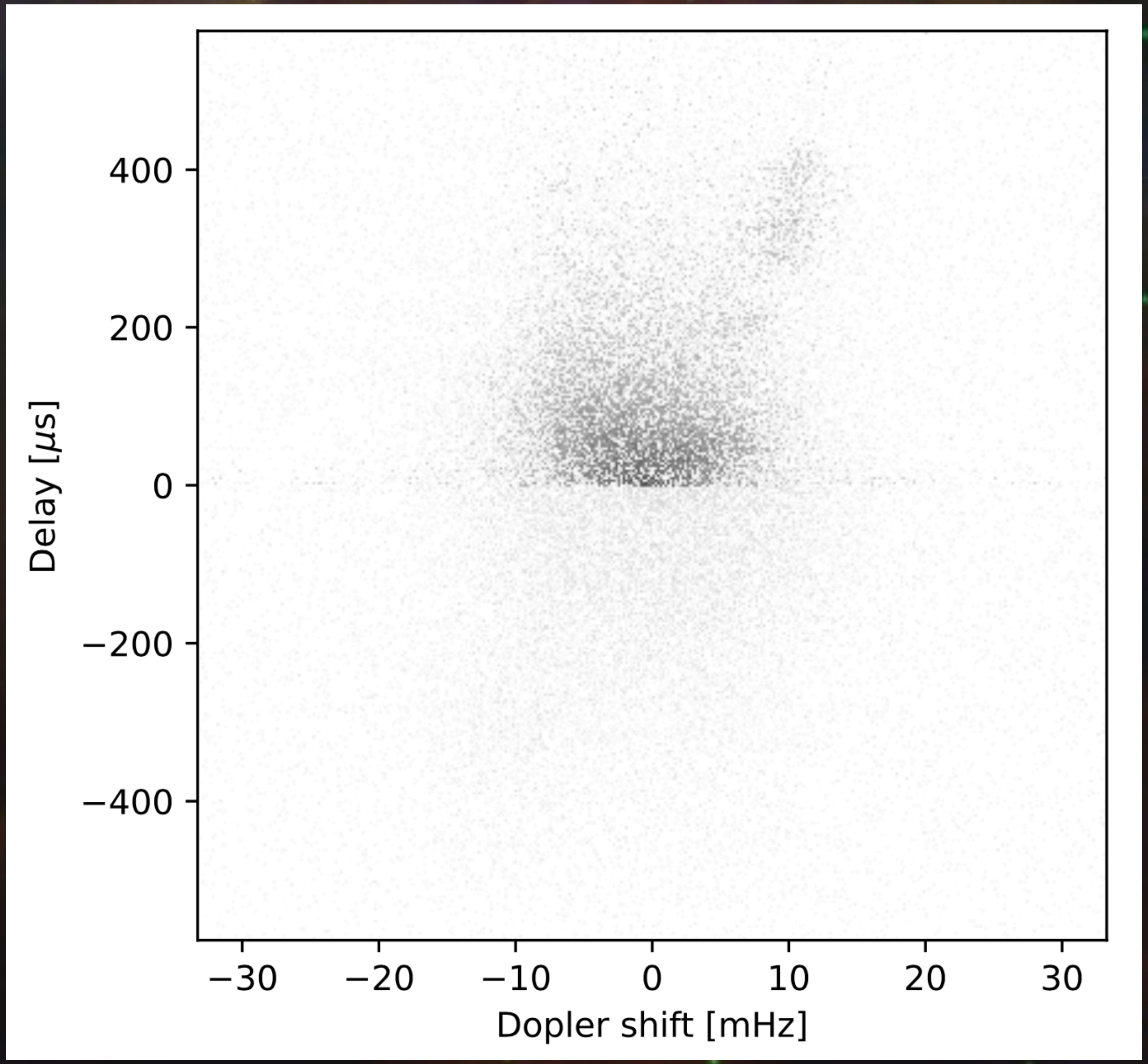
Mar

Results

Walker et al. 2013

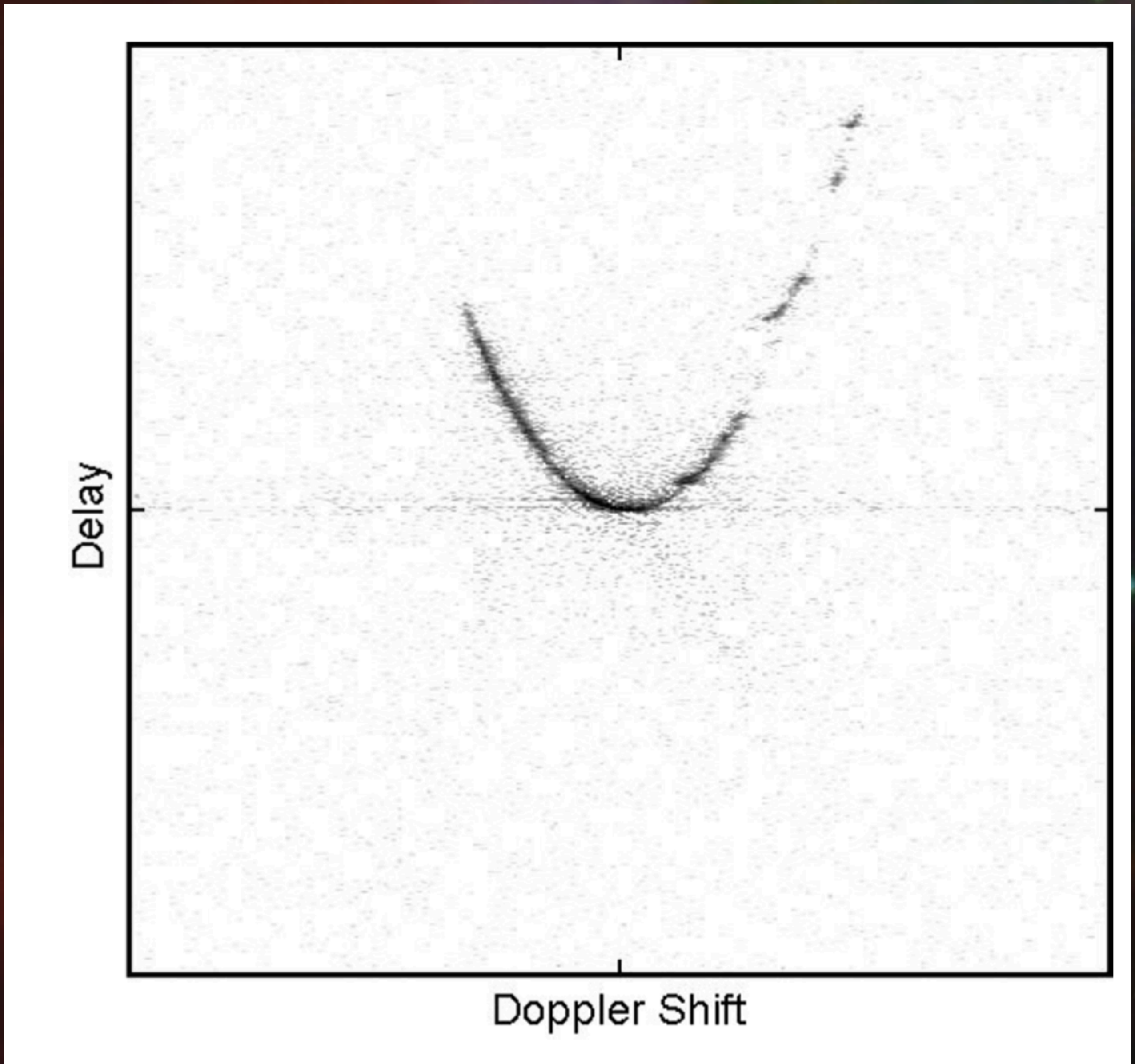
J1939+2134



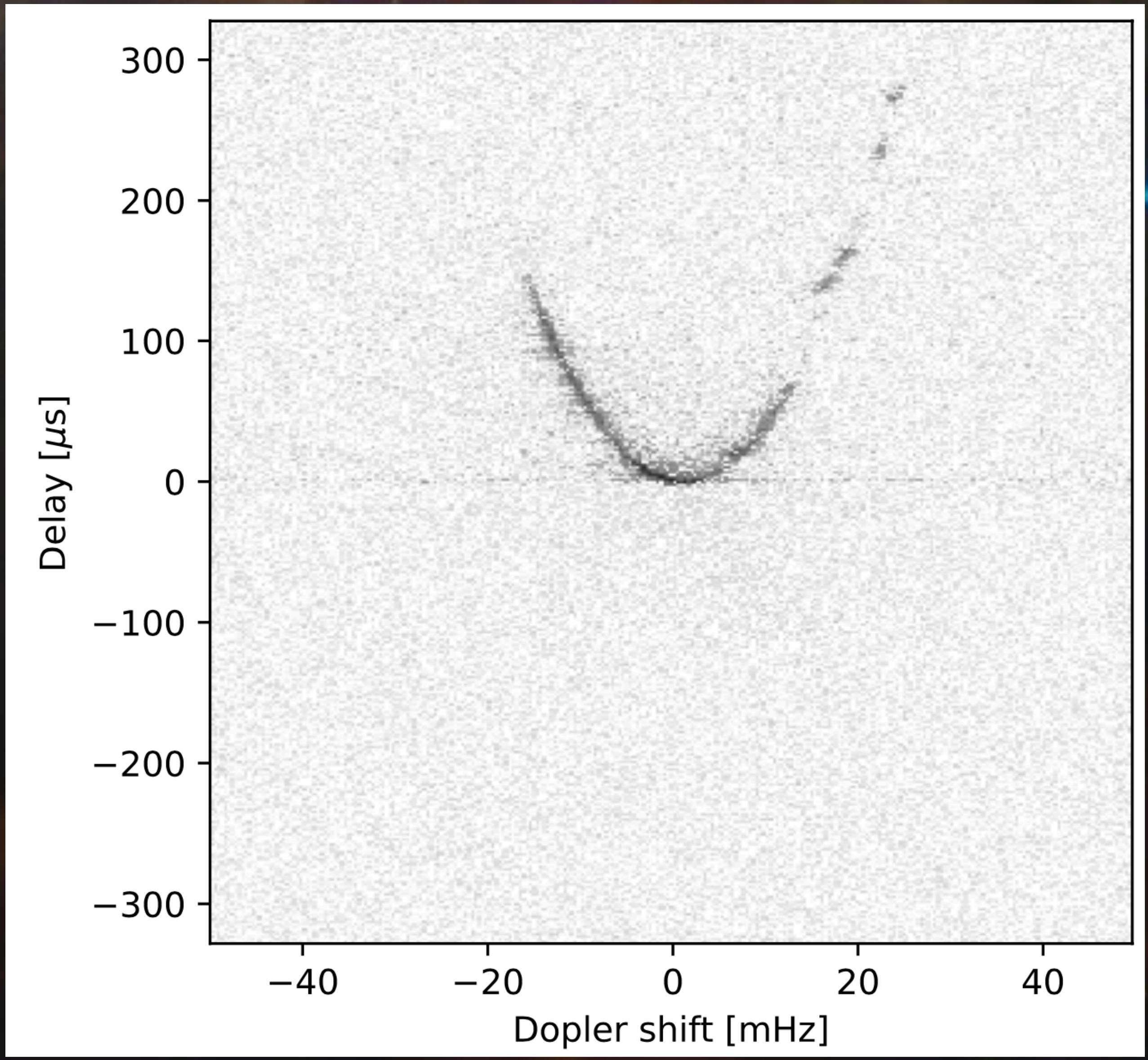


Results

Walker et al. 2008



J0837+0610



Manly Astrophysics