Holography via Dynamic Cyclic Spectroscopy

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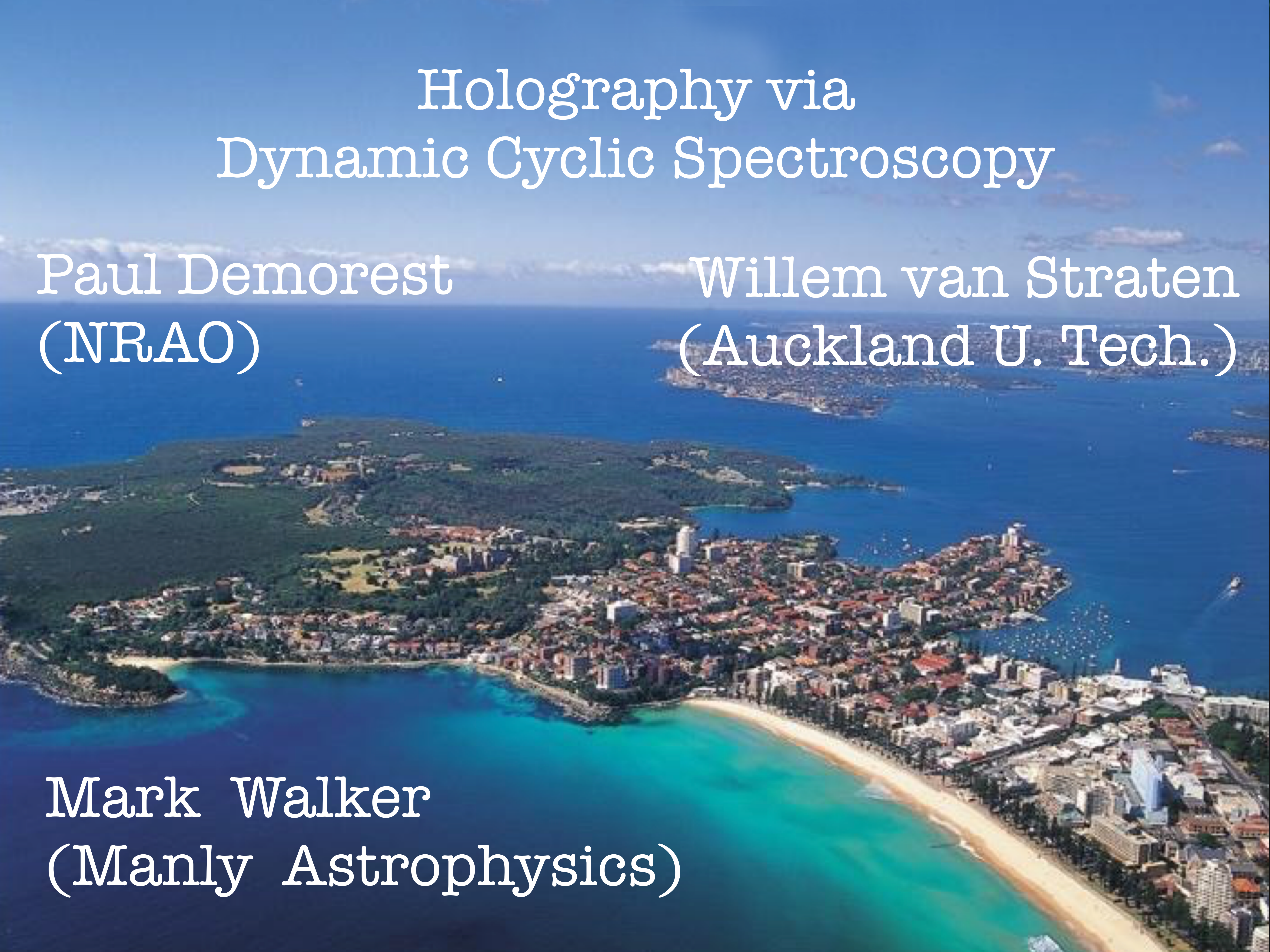
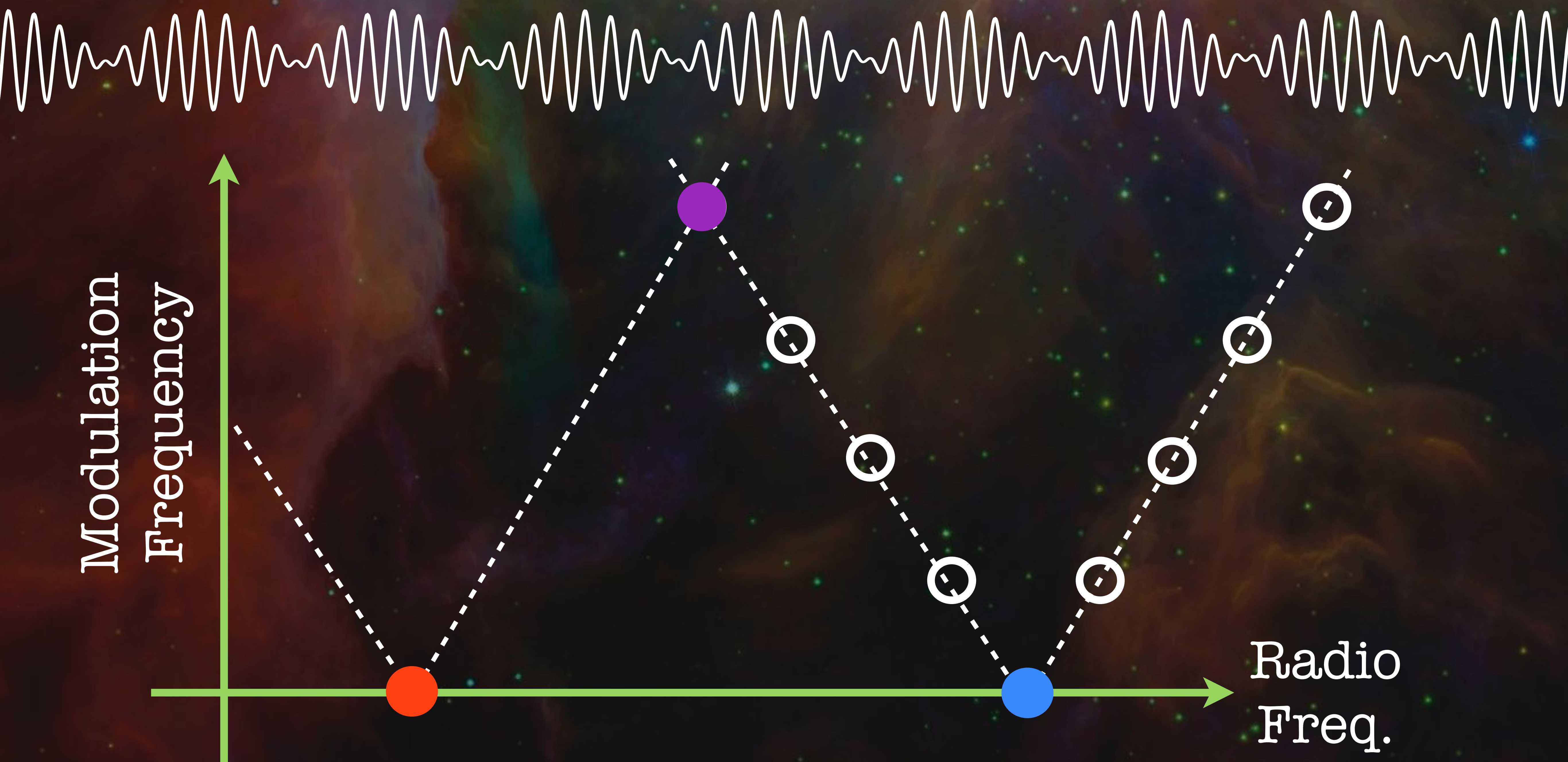


Overview

- Cyclic spectroscopy
- Phases and phase retrieval
  - Dynamic cyclic spectroscopy
- New approach to determining wavefield
  - Performance on dynamic spectra of B0834+06
  - Preliminary results for B1937+21 Dynamic CS

Cyclic Spectroscopy

(Demorest, 2011 MNRAS)

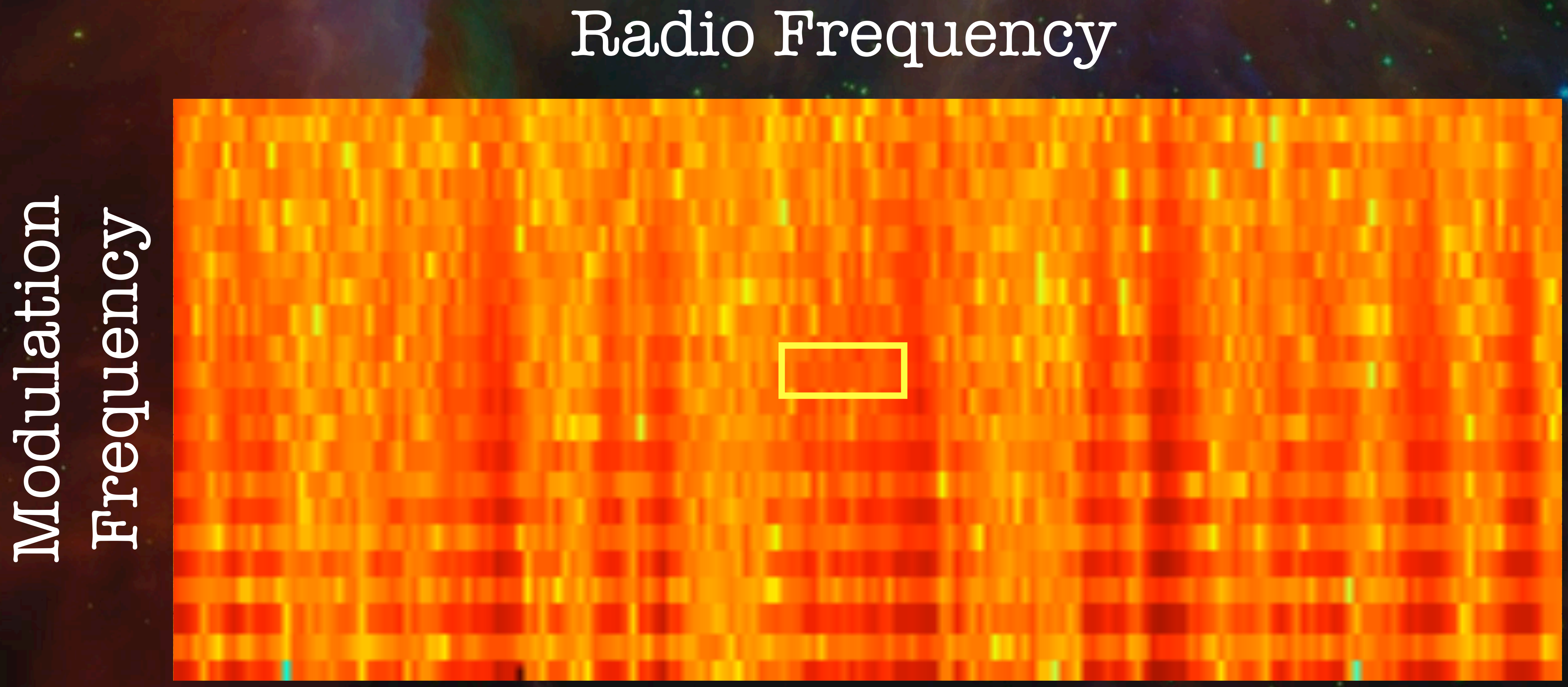


$$S_z(\alpha,\nu) = \langle Z(\nu+\alpha/2)\; Z^*(\nu-\alpha/2)\rangle$$

Manly Astrophysics

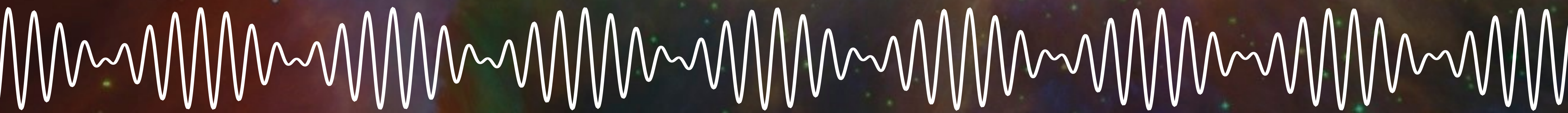
B1937+21 @ Arecibo, 428 MHz

(Demorest, 2011 MNRAS)



Cyclic Spectroscopy

(Demorest, 2011 MNRAS)



$$S_Z(\alpha,\nu) = \langle Z(\nu+\alpha/2)\, Z^*(\nu-\alpha/2)\rangle$$

Original signal: $X(\nu)$

Filtered signal: $Z(\nu) = H(\nu)\, X(\nu)$

$$S_Z(\alpha,\nu) = H(\nu+\alpha/2)\, H^*(\nu-\alpha/2)\, S_X(\alpha,\nu)$$

$$S_X(\alpha)$$

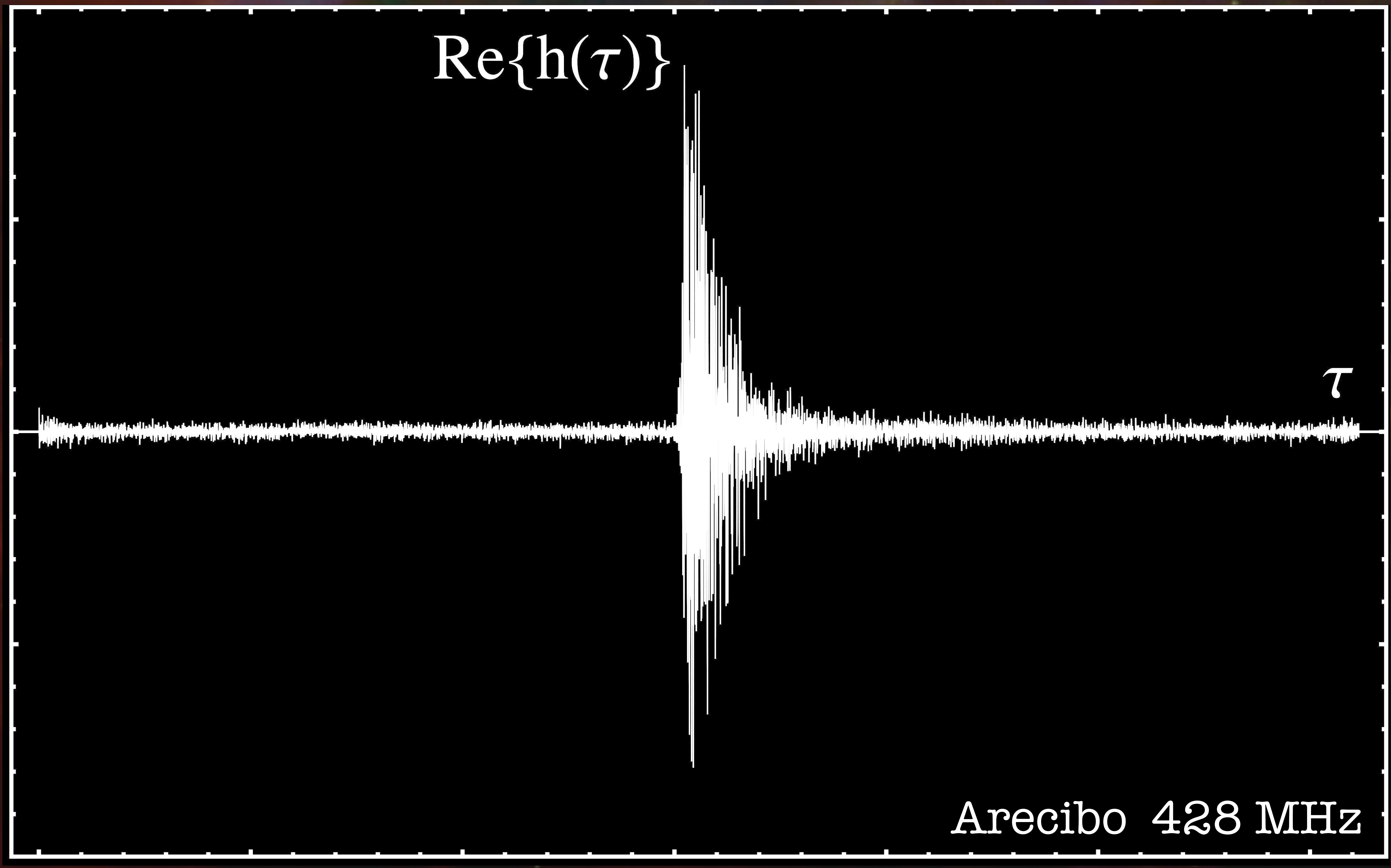
FT(pulse profile)

$$H(\nu) \longleftrightarrow h(\tau)$$

Filter / Wavefield / Impulse Response

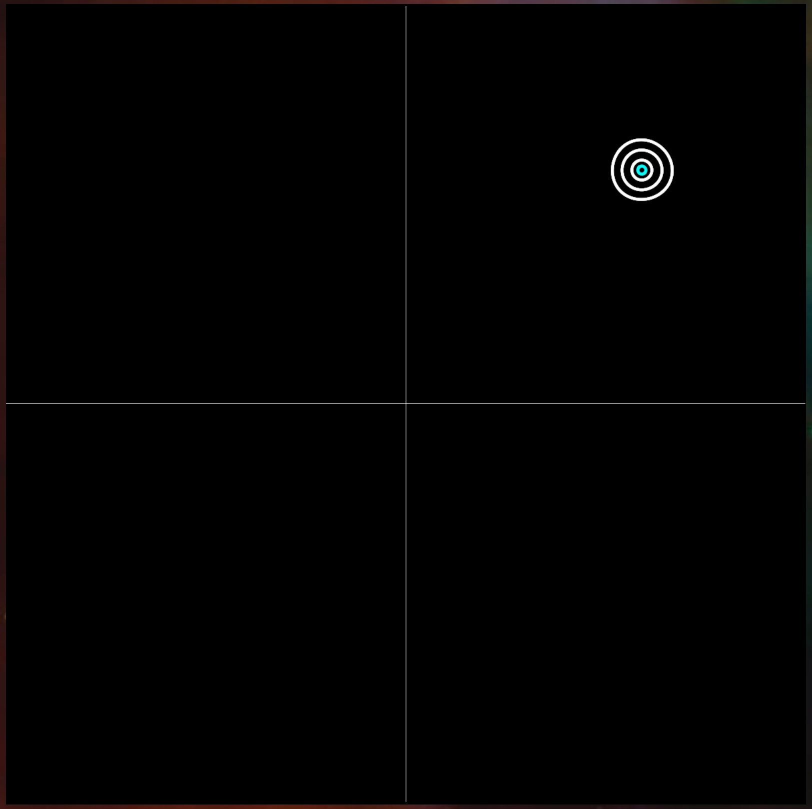
Example impulse response for B1937+21

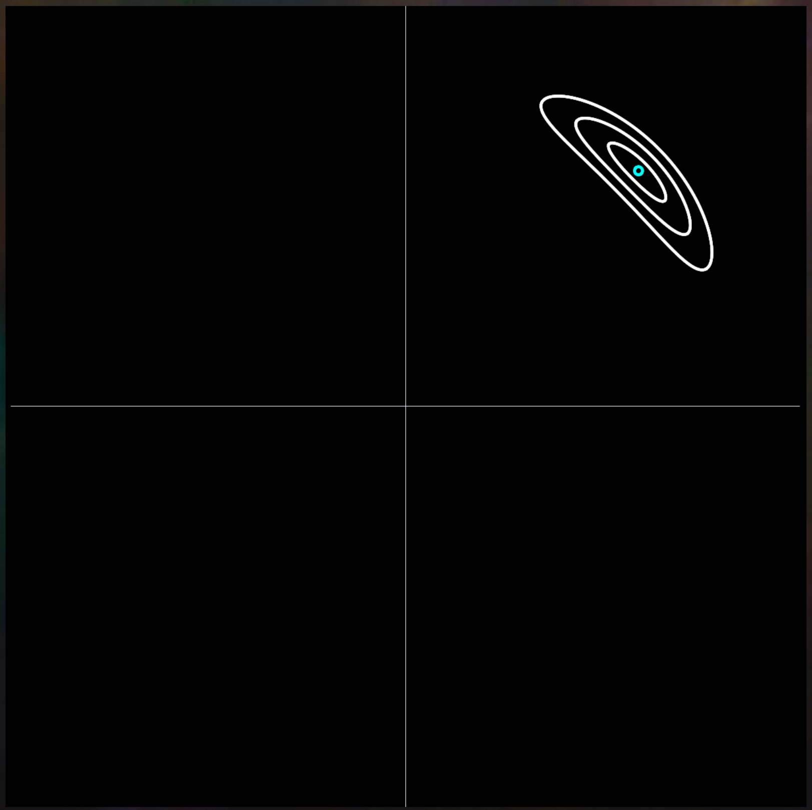
(MW, PD & WvS, 2013 ApJ)

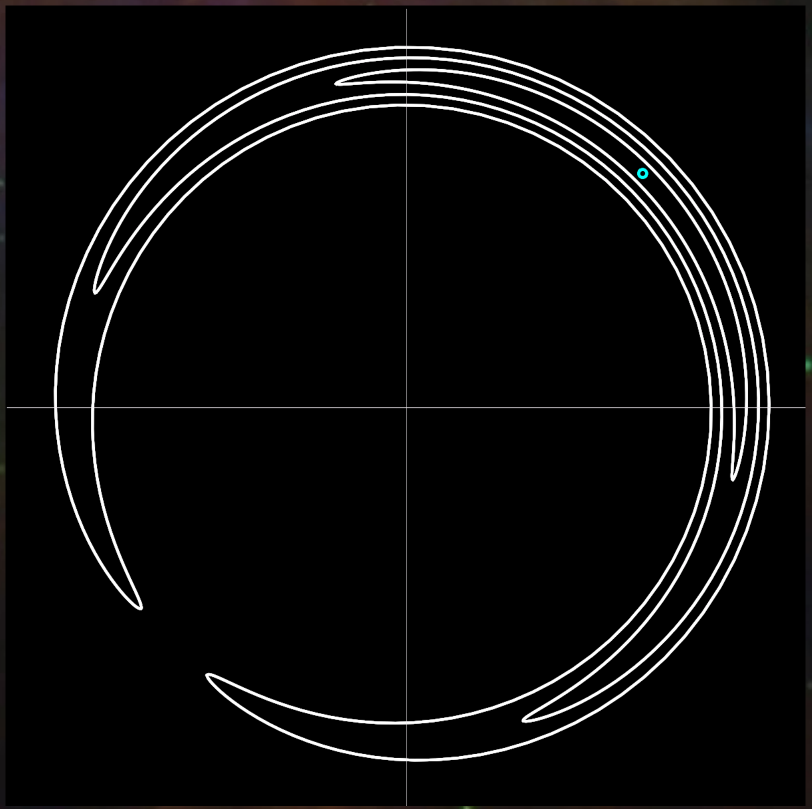


Problem: phase noise on h(τ)

(Dan Stinebring, 2013, priv. comm.)







$\tau \gg$ Pulse Width $\quad$ $\tau \sim$ Pulse Width $\quad$ $\tau \ll$ Pulse Width

Cyclic Spectroscopy provides some direct phase information

But still necessary to infer ("retrieve") information on the phase structure of h from the cyclic spectrum amplitudes.

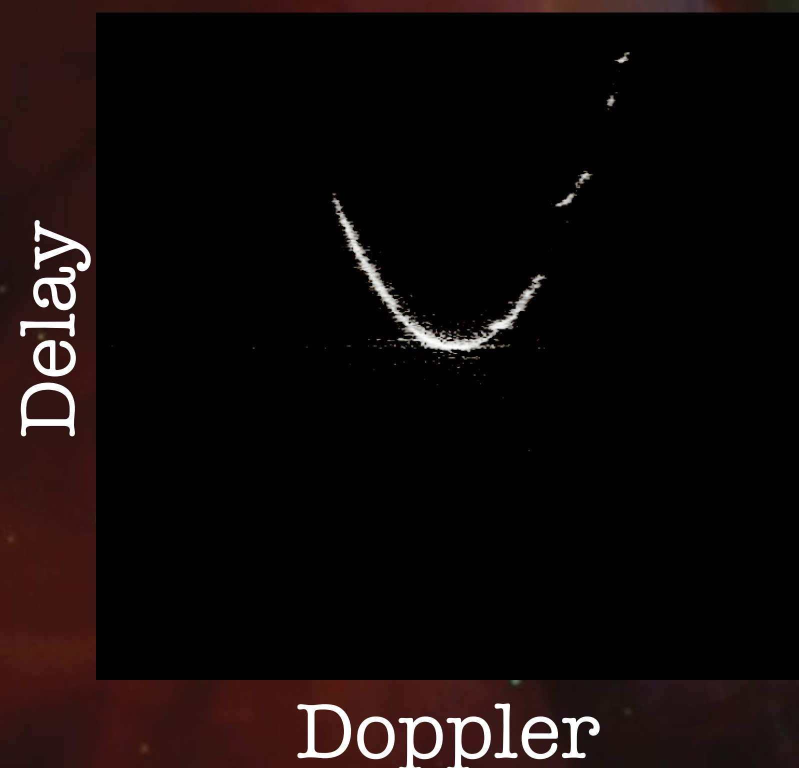
Example wavefield for B1937+21

(MW, PD & WvS, 2013 ApJ)

Fourier Relationships

Wavefield

$h(\tau,\omega)$

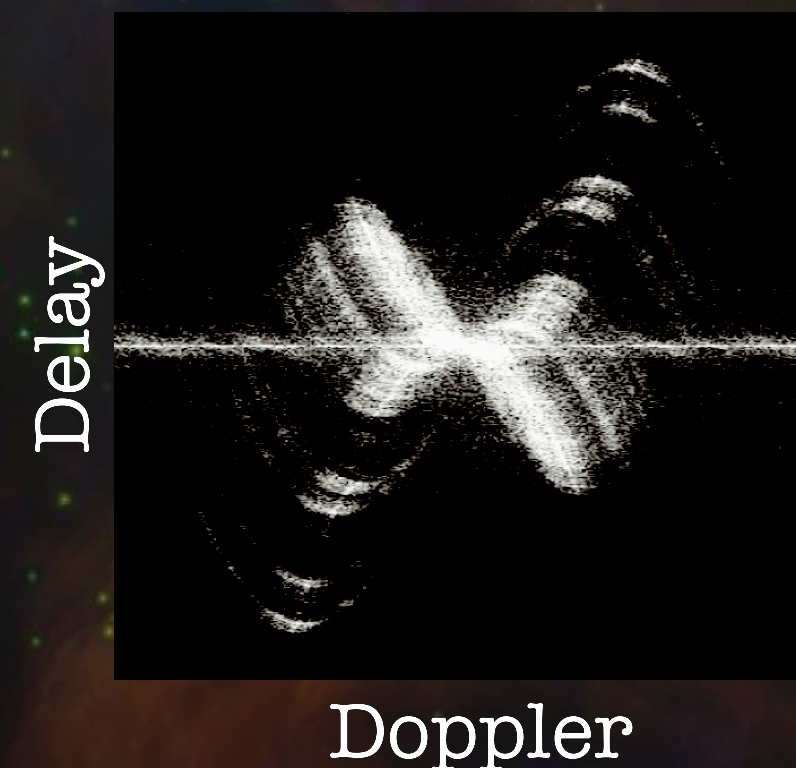


Dynamic Spectrum

$I(\nu,t)=|H(\nu,t)|^2$

Secondary Spectrum

$S(\tau,\omega)=|I(\tau,\omega)|^2$



Phase retrieval

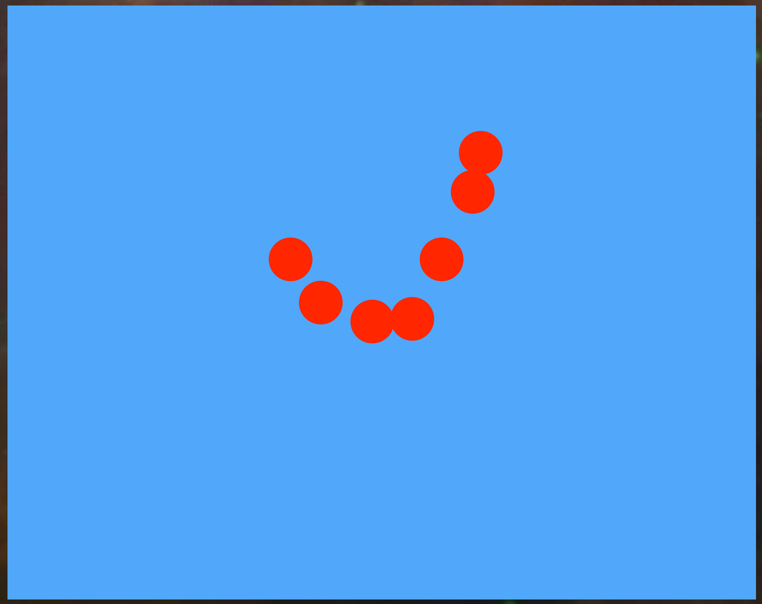
- Usually cannot retrieve phases in one dimension
  - Need to solve for $H(\nu, t)$, not individual $H(\nu)$
- Requires many more constraints than unknowns
  - Sparse solution (or tight support constraint)
- Most commonly used method is HIO (Fienup 1982)
  - Iterative projections + support constraint
    - Unclear how to incorporate phase information
- Some success with CLEANing (MW++ 2008)
  - But slow and unreliable
- New method: “Wirtinger Flow” (Candes++ 2015)
  - (Wirtinger) gradient descent of $\sum|\text{error}|^2$
    - Large signal spaces are manageable

Our approach

- Want a sparse solution $\therefore$ minimise $\sum|\text{error}|^2 + \lambda|h|$ using Proximal Gradient method
    - Iterative Shrinkage Thresholding Algorithm (ISTA)
        - Wirtinger gradient, because h is complex
- FISTA = Fast ISTA (Beck & Teboulle 2009)
    - ISTA with Nesterov-style acceleration
    - Guaranteed rapid convergence on convex problems
        - Guaranteed convergence on non-convex problems
- Needs a de-bias step to achieve high dynamic range
- Build up model wavefield using FISTA repeatedly, with progressively smaller λ
    - Care needed to separate h from h^* ("twin" image)

Hierarchical FISTA

Initialise h, λ



FISTA Iterations — Minimise $\sum|\mathrm{error}|^2 + \lambda|h(\overline{\mathrm{support}})|$

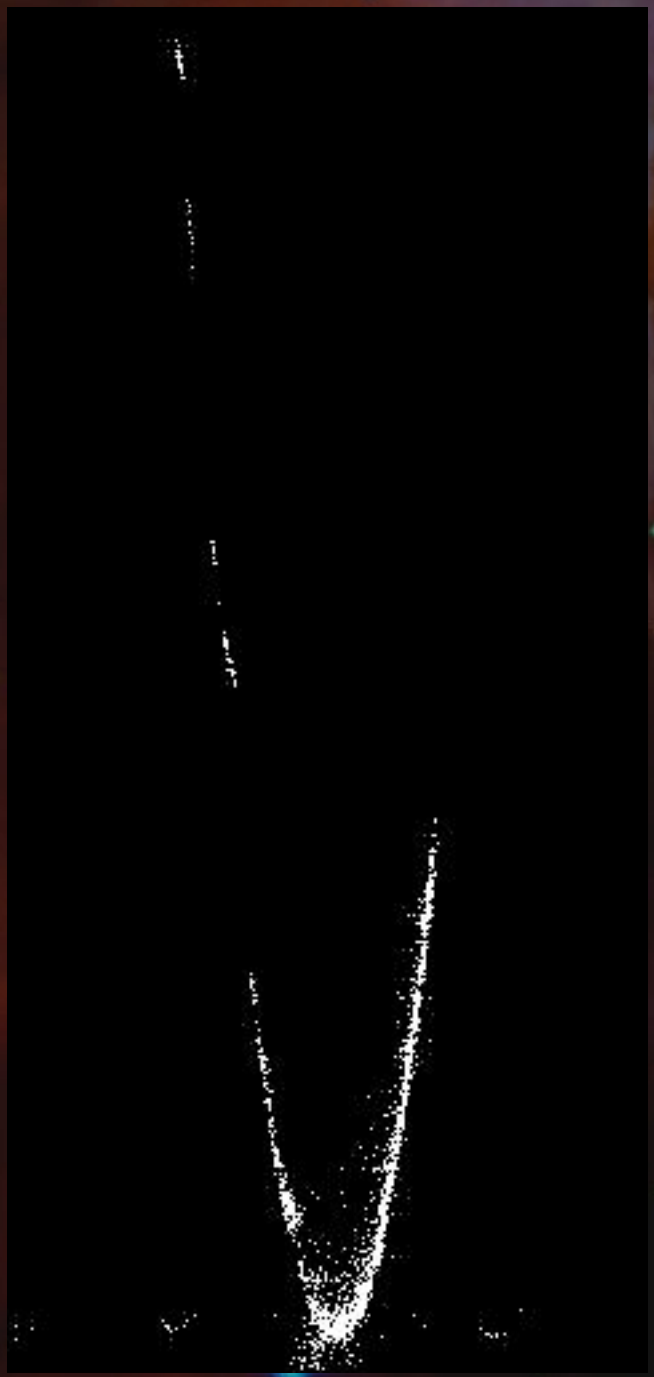
Hard Threshold — Remove small components in support

Refresh Support

Decrease λ

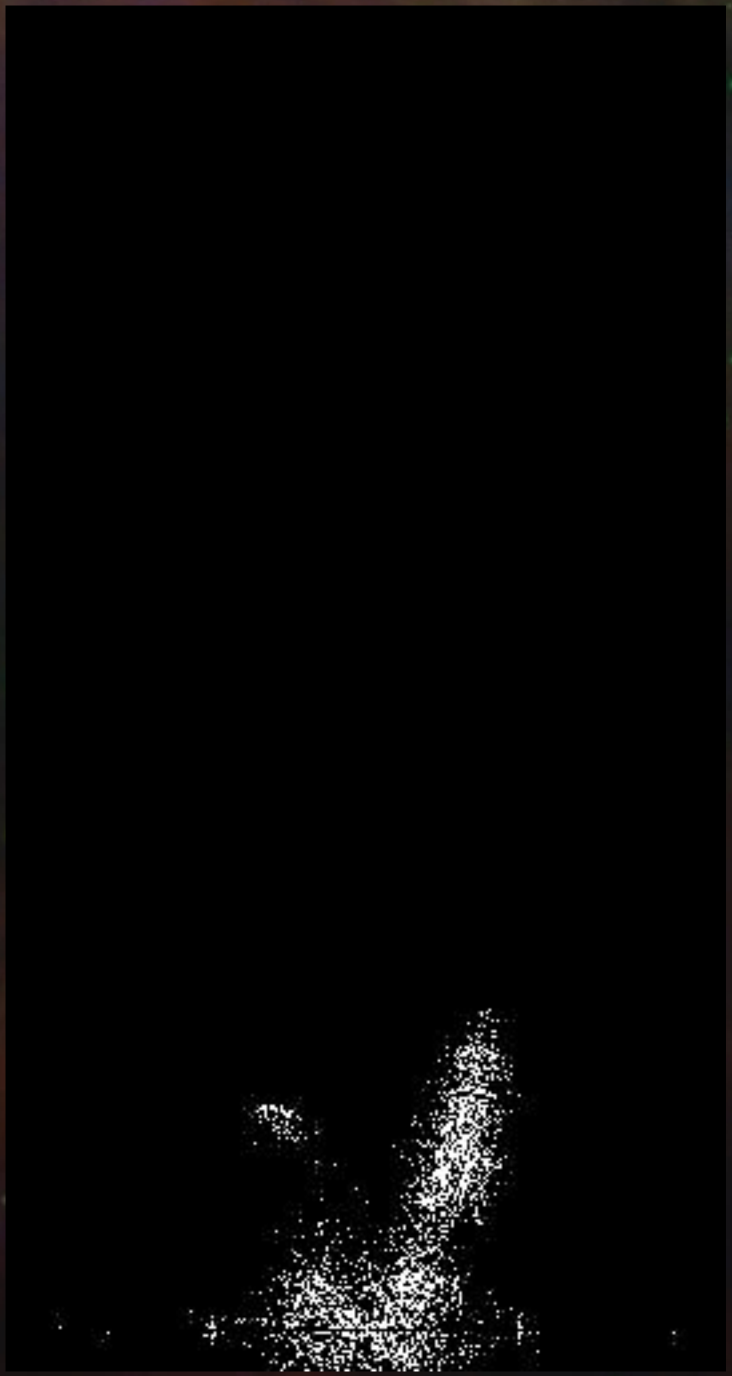
Results on B0834+06 dynamic spectra

(Data courtesy Dan Stinebring)



MJD53014

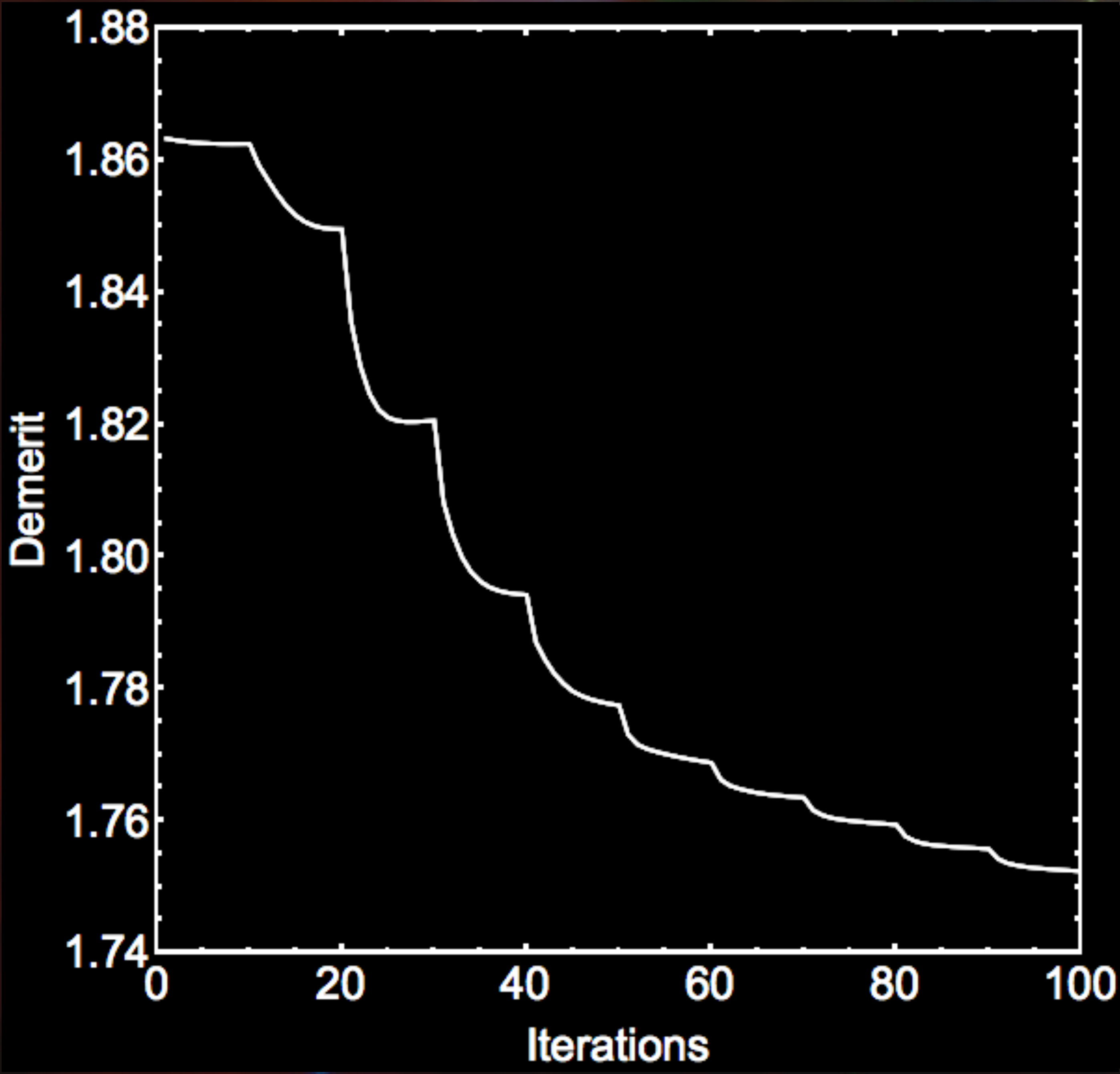
MJD53006

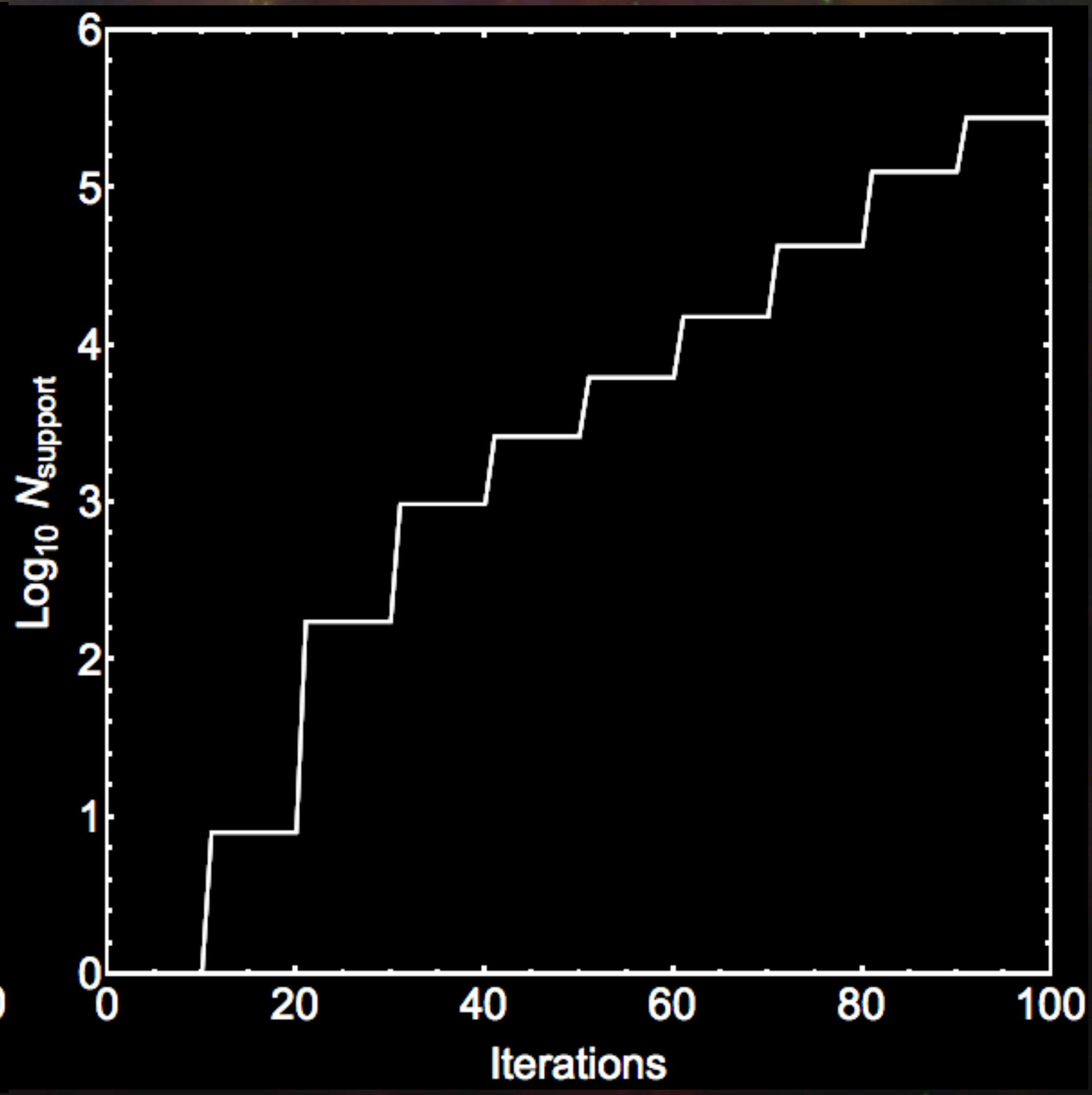


MJD53023

Preliminary results for B1937+21

Dynamic Cyclic Spectra (78 temporal samples)





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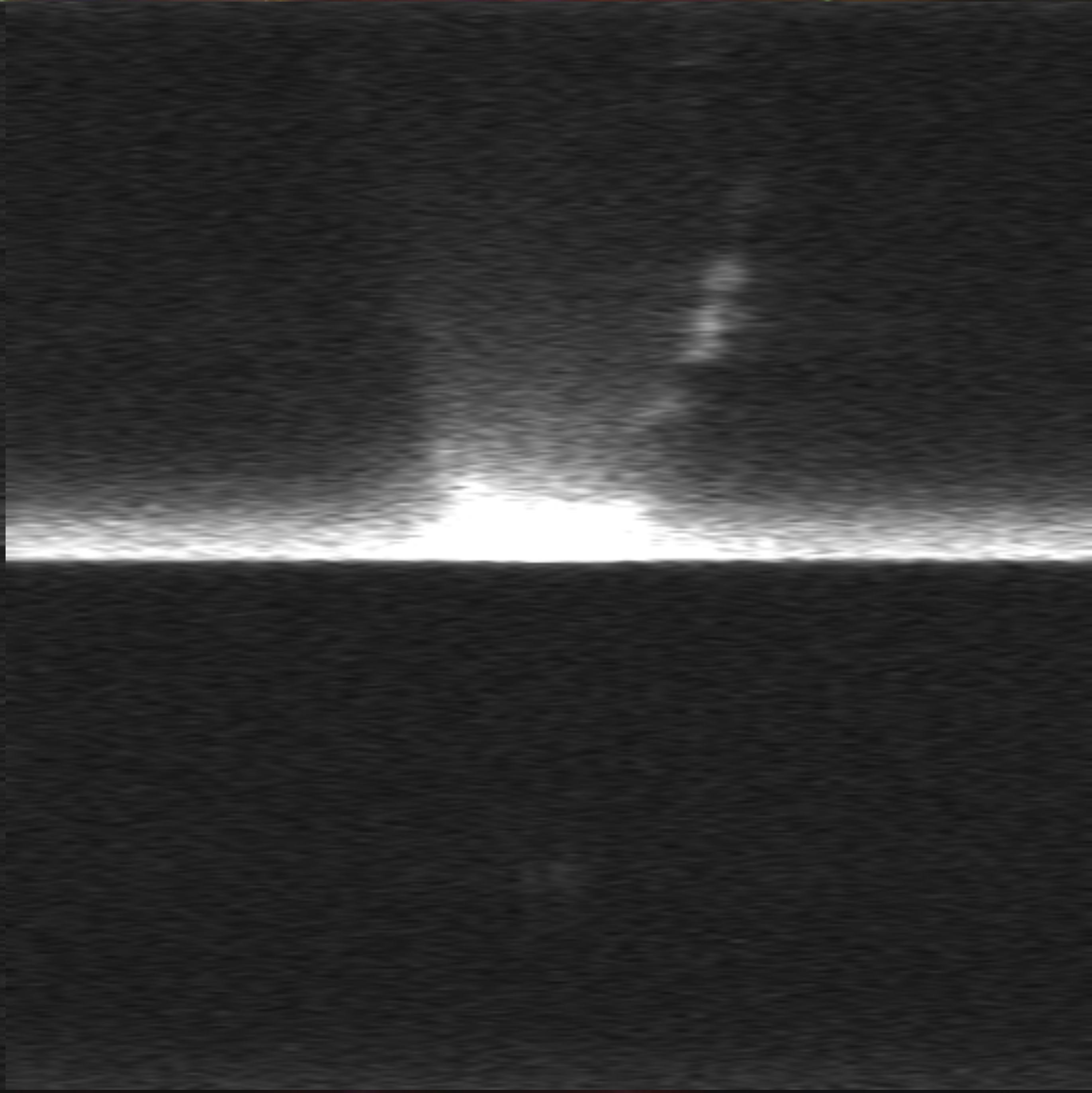

Individual Samples

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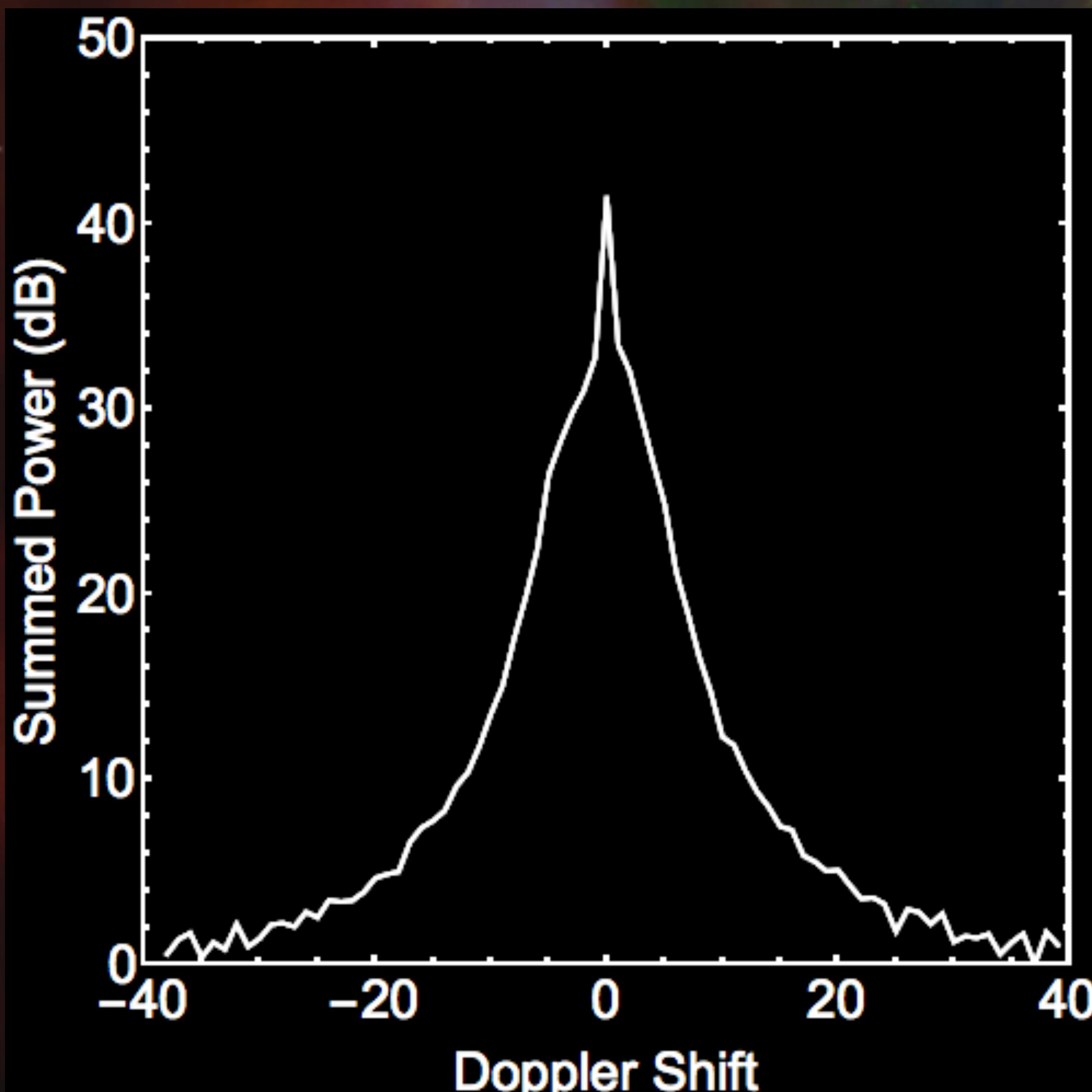
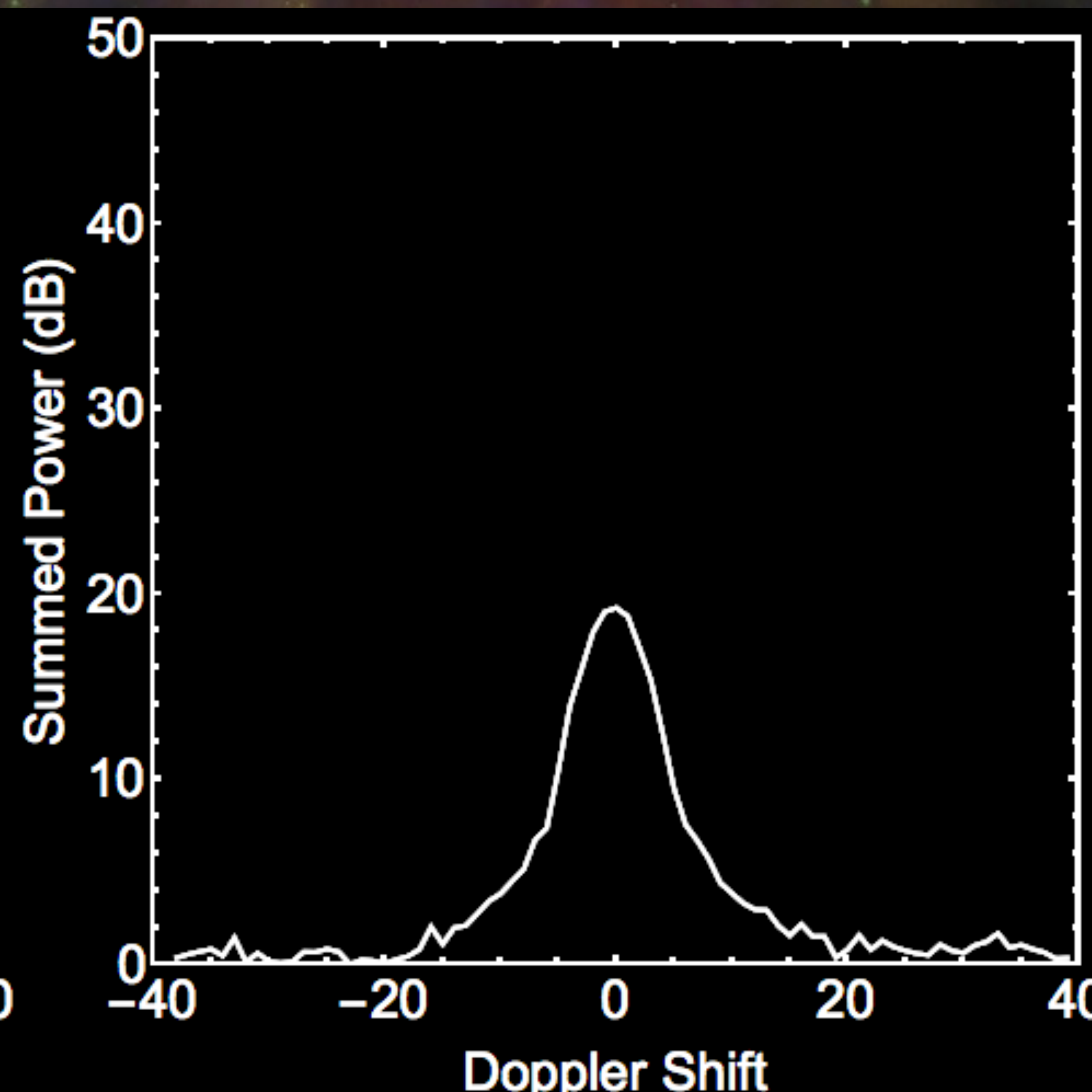


Hierarchical FISTA



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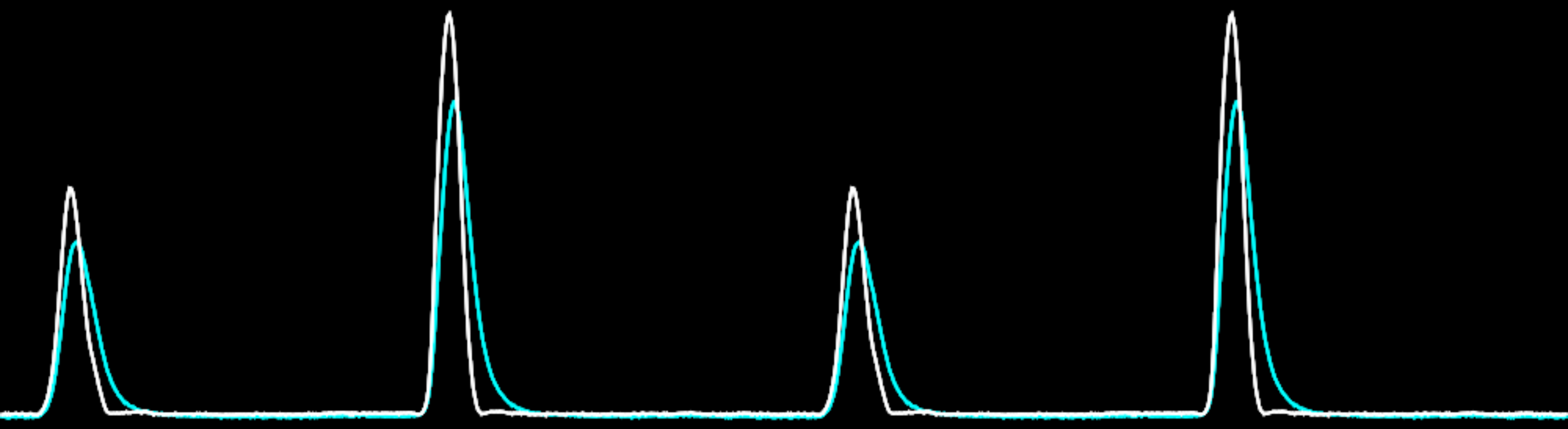

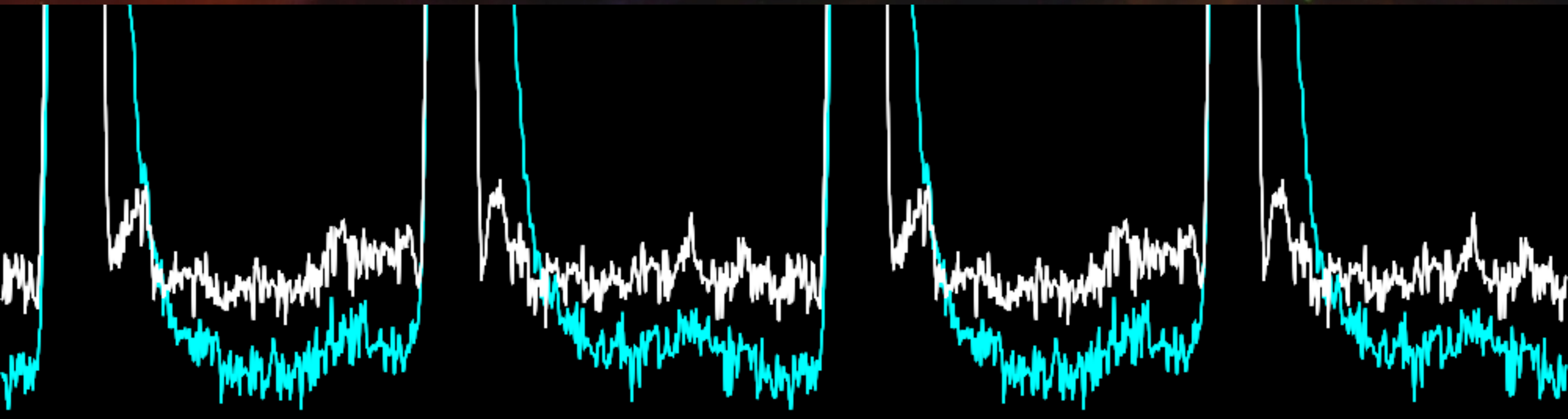

Where to from here?

- Improve reliability of phase retrieval
  - Better handling of intrinsic pulsed flux variations
- Python implementation (currently Mathematica)
- Run on cluster (currently laptop)
- Performance tests on dynamic cyclic spectra of
  - B1937+21 (MSP) and B0834+06 (Slow PSR)
- Add options for basis functions (e.g. wavelets)
- Extend to full Stokes