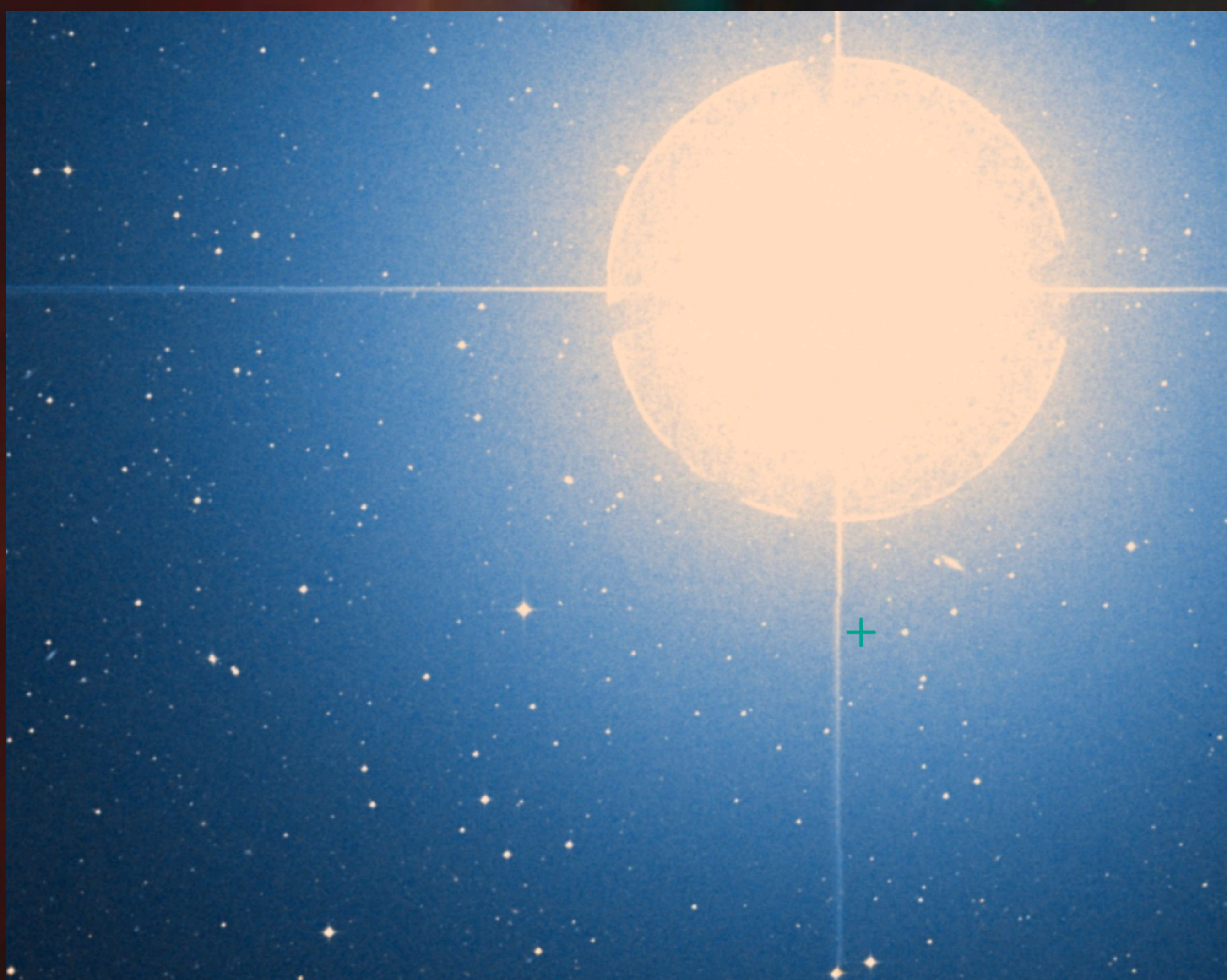
# Caustics in Plasma Lensing

Artem Tuntsov (Manly Astrophysics)



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CSIRO Marsfield,
CSIRO Kensington)

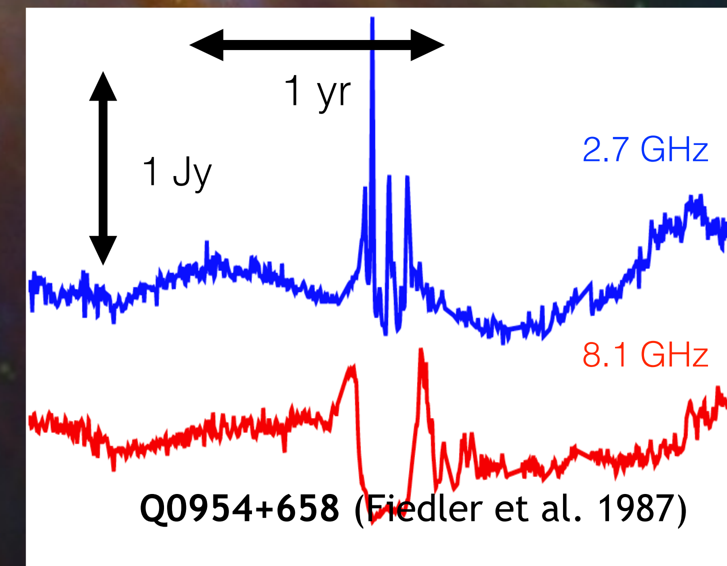
Gravity meets Plasma, Kunming

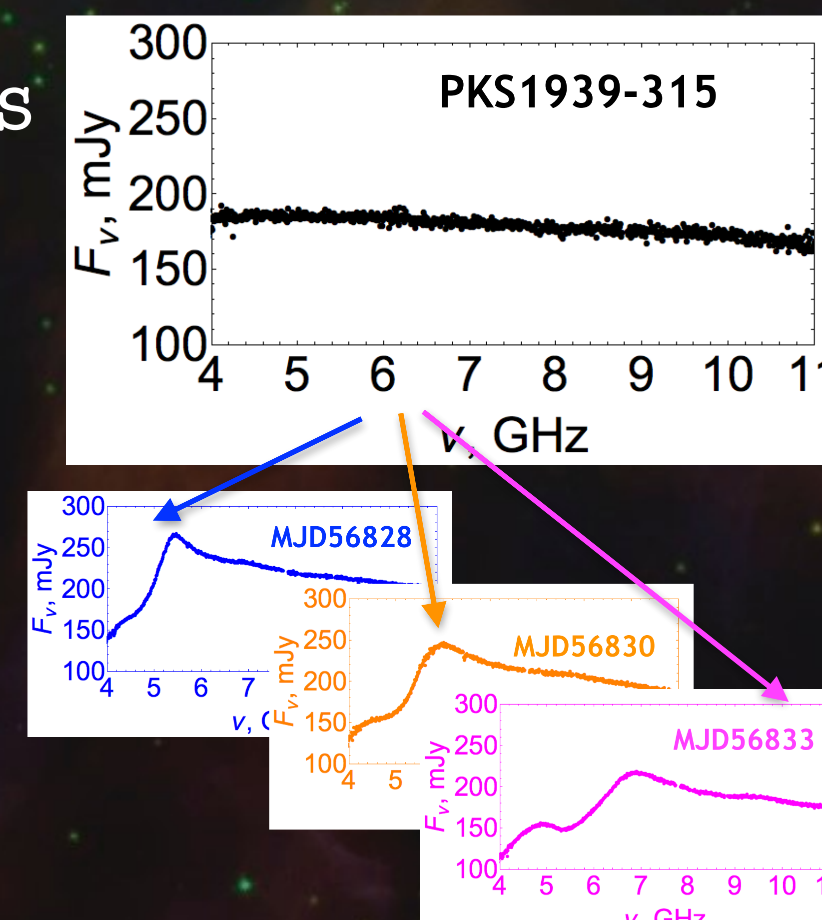
2019/08/20

# Plasma lenses in radio spectra

ATESE/TAILS: impossible plasma lenses

- Seen in some compact sources
- Efficiently discovered in spectra
- Come in 2.5 flavours (grrr!?)
  - slow: extreme scattering events, ESEs
  - fast: (extreme) intra-day variables, (x)IDVs
  - faster: hours, IHVs (close screens)
- Problematic plasma physics
- Wealth of data in dynamic spectra
  - not just fitting model parameters
  - actually reconstructing the lens!



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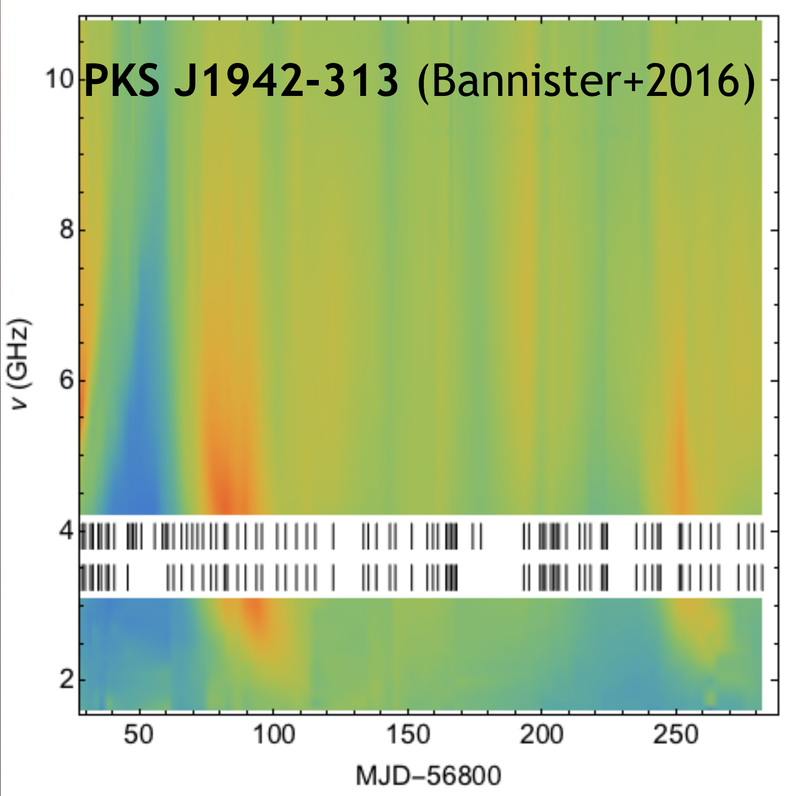
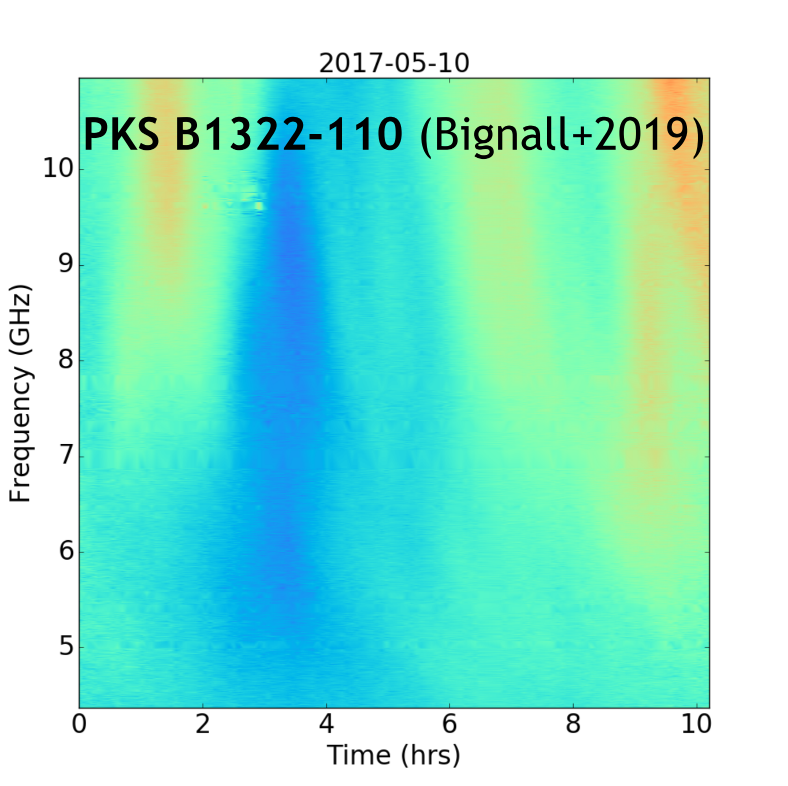


# Assumptions & conventions

Keep it simple

- no gravity
- no polarisations

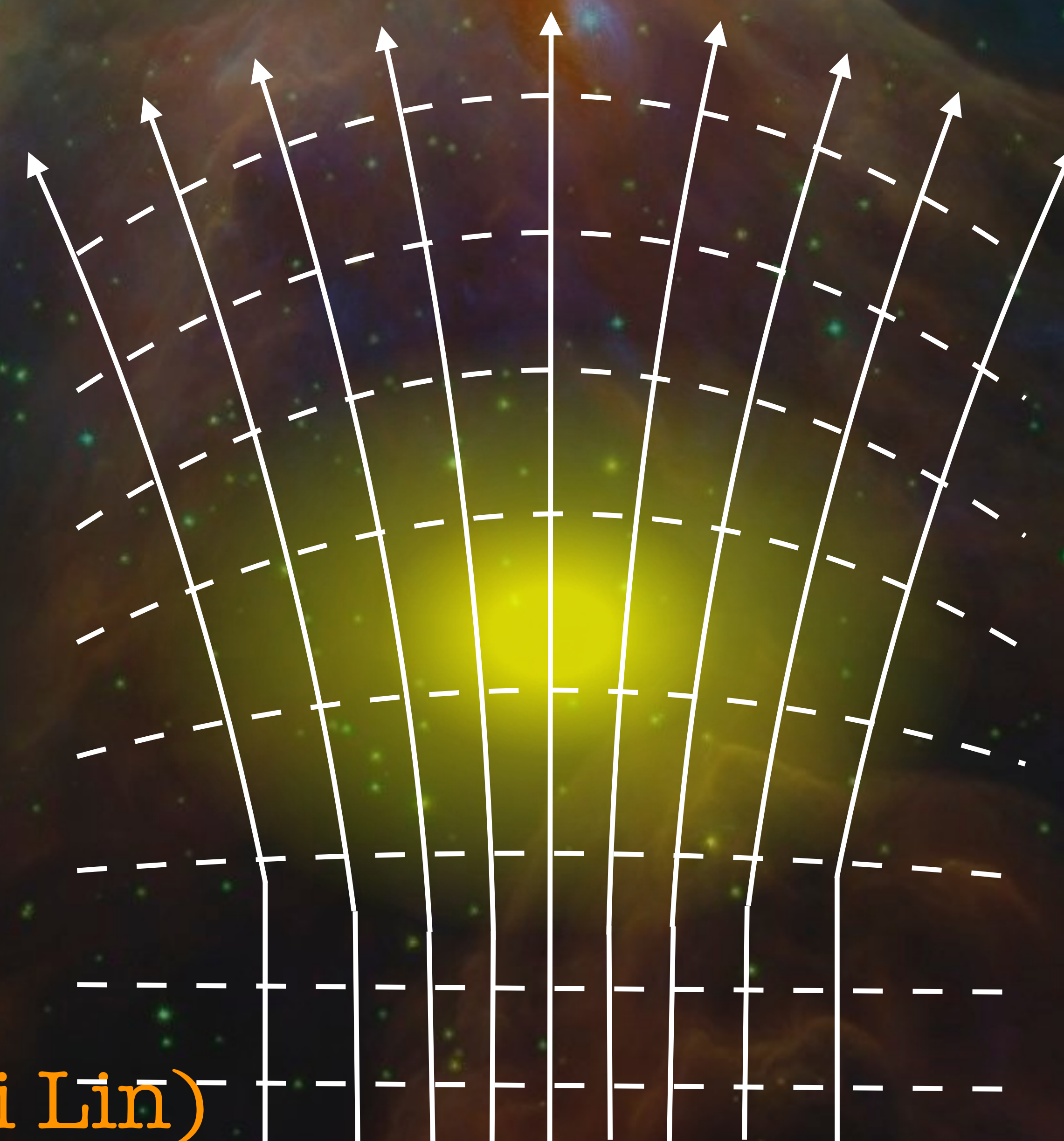
Single thin screen

- above plasma frequency
- small deflections (Born)
- electrons advance phase

No diffraction/coherence (Fangxi Lin)

- geometric (ray) optics
- no interference b/w rays

Point source



$$\Phi = -\lambda r_e N_e$$

$$\overline{\alpha} = k^{-1}\overline{\partial}\Phi \propto \lambda^2 \overline{\partial} N_e$$

# Observables and where they live

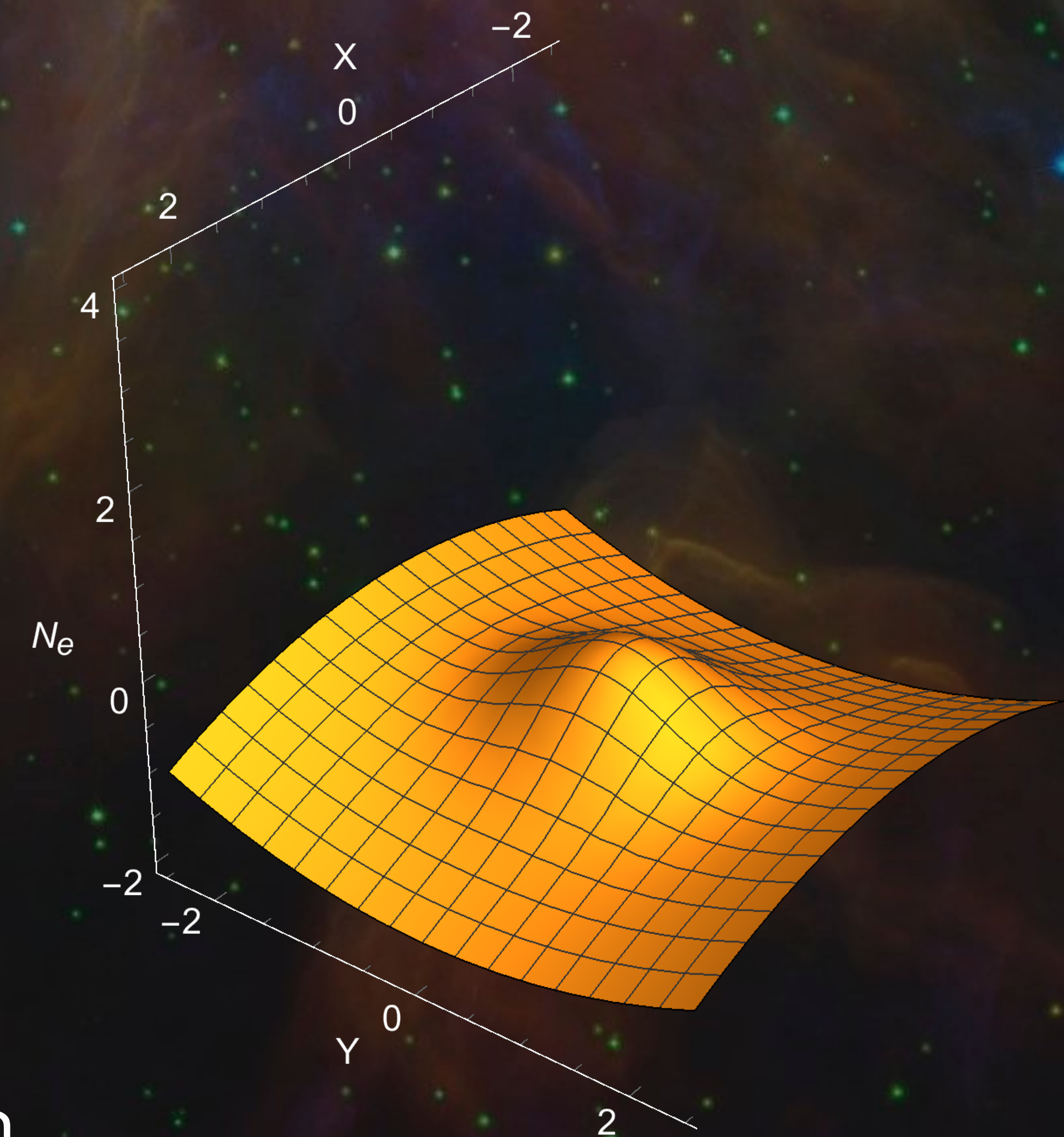
Frequency dependent lens map

$$\overline{\alpha} : \overline{\theta} \to \overline{\beta} = \overline{\theta} - \lambda^2 \, \overline{\alpha}(\overline{\theta})$$

- arbitrary (!) gradient field
- all $\boldsymbol{\alpha}$,κ,γ... quoted at λ=1

Now ($\boldsymbol{\beta}$, $\lambda^2$) space looks hairy!

- bristles curve for λ or ν axes
- not a proper fibre bundle




We observe flux ∝ magnification μ

- (exc. images wonder w/ VLBI)
- known μ(λ) along each bristle

Handy for $\boldsymbol{\beta}$ to be observer position

- think of projected pattern

Flux is sampled on data cylinder

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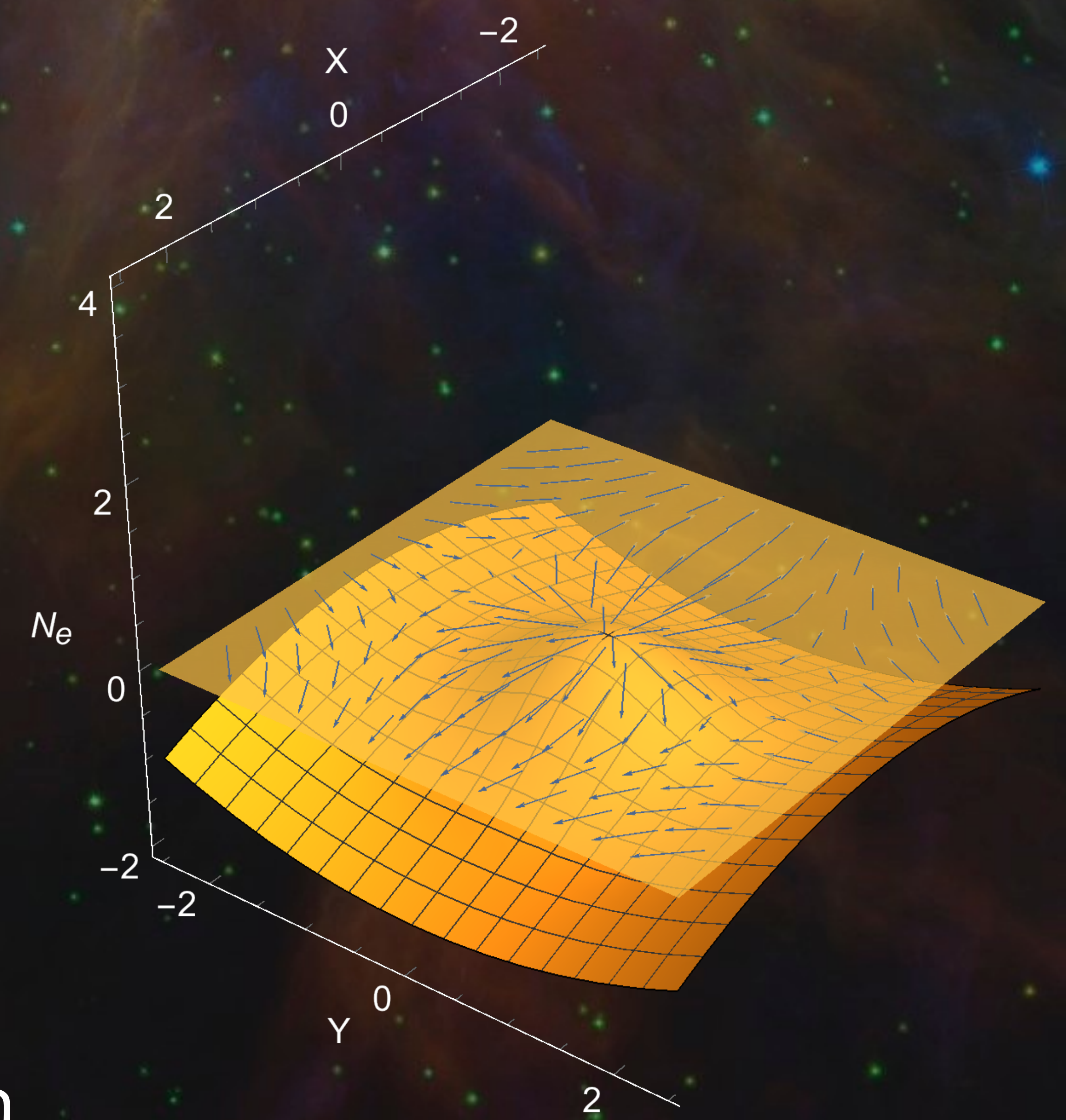

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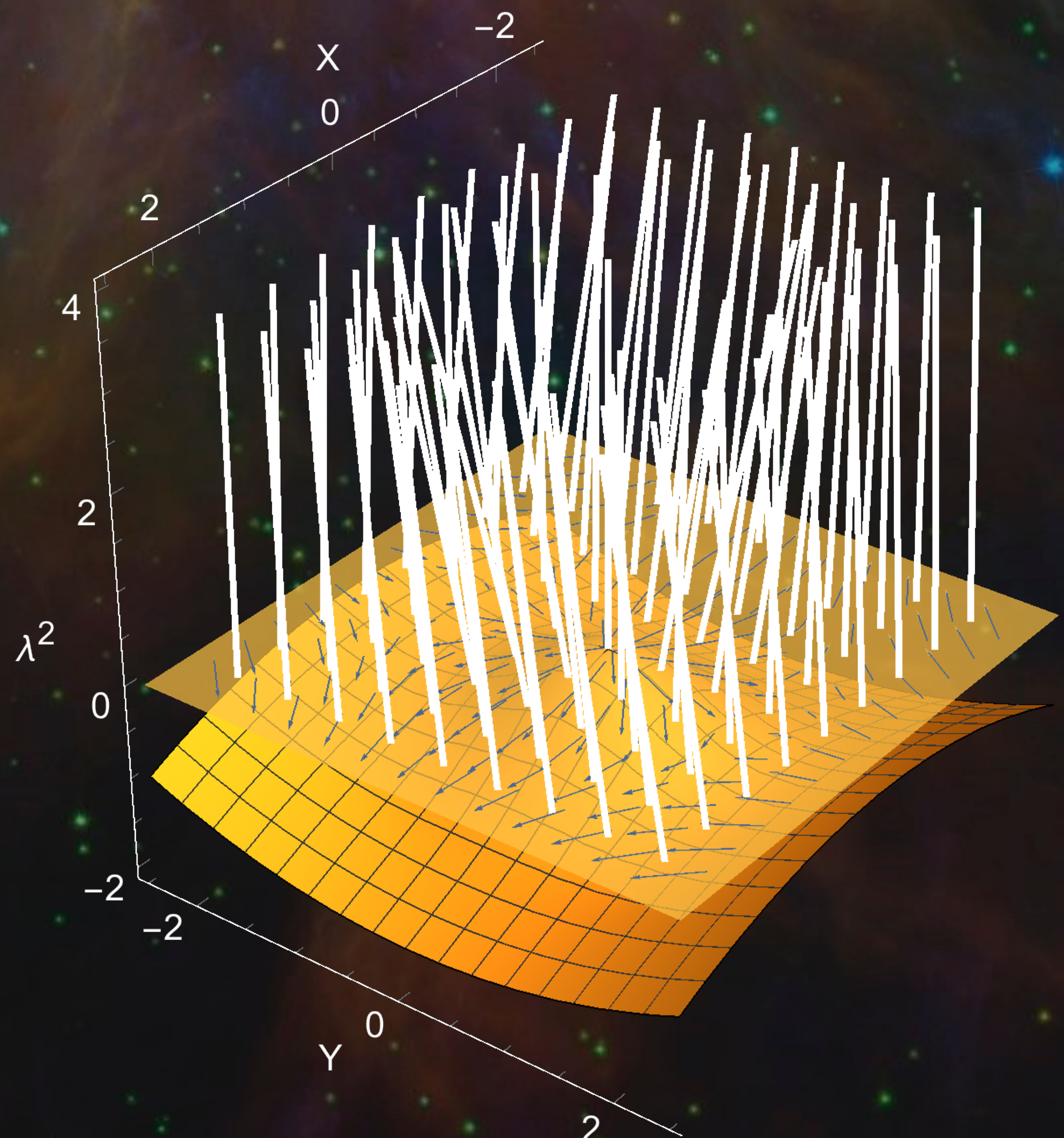


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$$\mu(\overline{\theta}, \lambda) = \left| \det\left(1 - \lambda^2 \partial_{\overline{\theta}} \overline{\alpha}\right) \right|^{-1}$$

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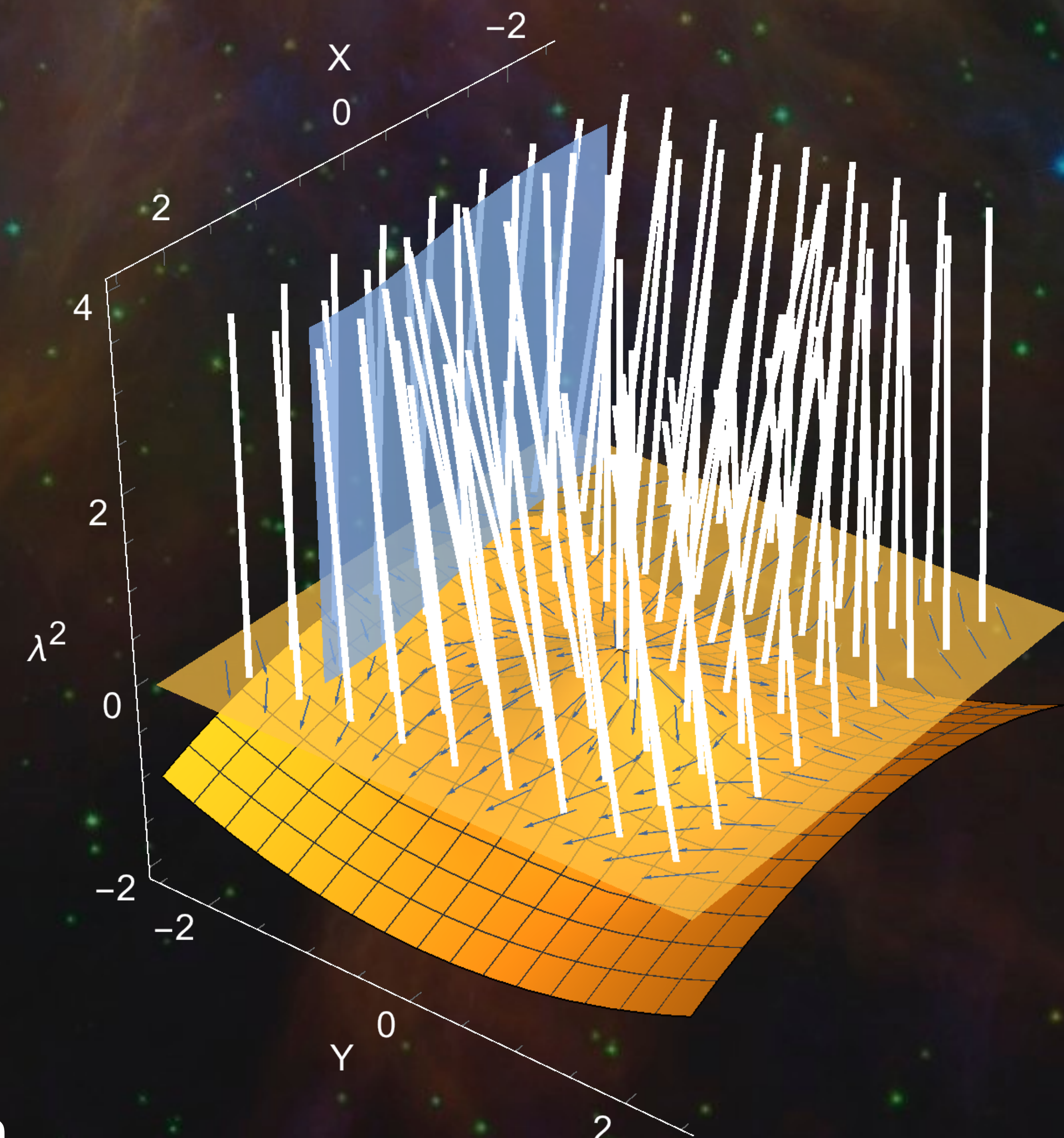


We observe flux $\propto$ magnification $\mu$

- (exc. images wonder w/ VLBI)
- known $\mu(\lambda)$ along each bristle

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# Reconstructing phase screen

Problem:

$F_\nu(t, \nu) \rightarrow \mu[\boldsymbol{\beta}(t), \lambda] \rightarrow \boldsymbol{\alpha}(\boldsymbol{\theta}) \rightarrow N_e(\boldsymbol{\theta})$

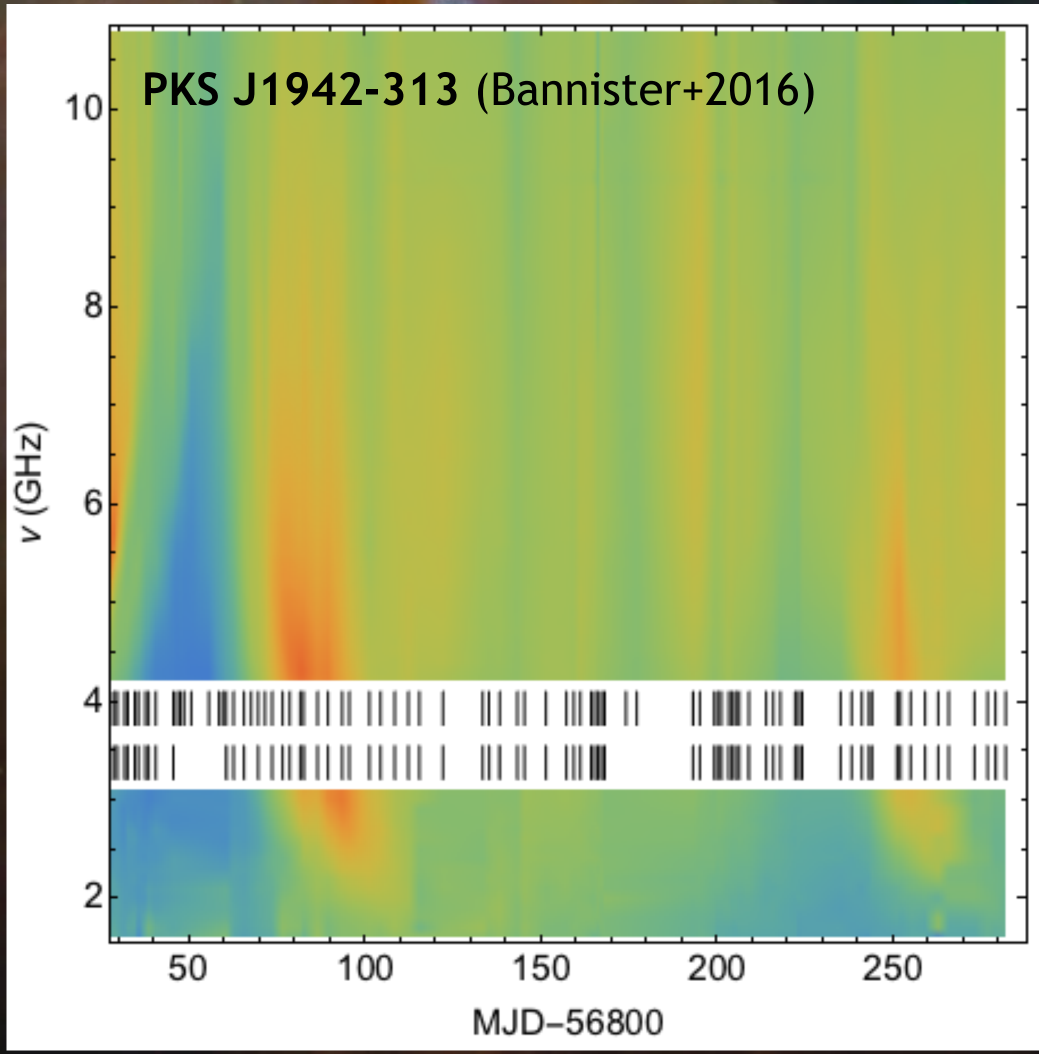
Good news: 2D solution exists

- e.g., X-disperse: $\boldsymbol{\alpha}(\boldsymbol{\theta})=\lambda^{-2}[\boldsymbol{\theta}-\boldsymbol{\beta}(\boldsymbol{\theta})]$

$$\theta_2 = A\lambda^2, \ \ \theta_1 = \int_0^\beta \mathrm{d}\beta' \left[ \pm\mu\left(\beta', \sqrt{\theta_2/A}\right) - 1 \right]$$

Bad news: not what we need

- ill-posed (non-unique)
- unphysical (e.g., A>>1)



# Reconstructing phase screen: other way

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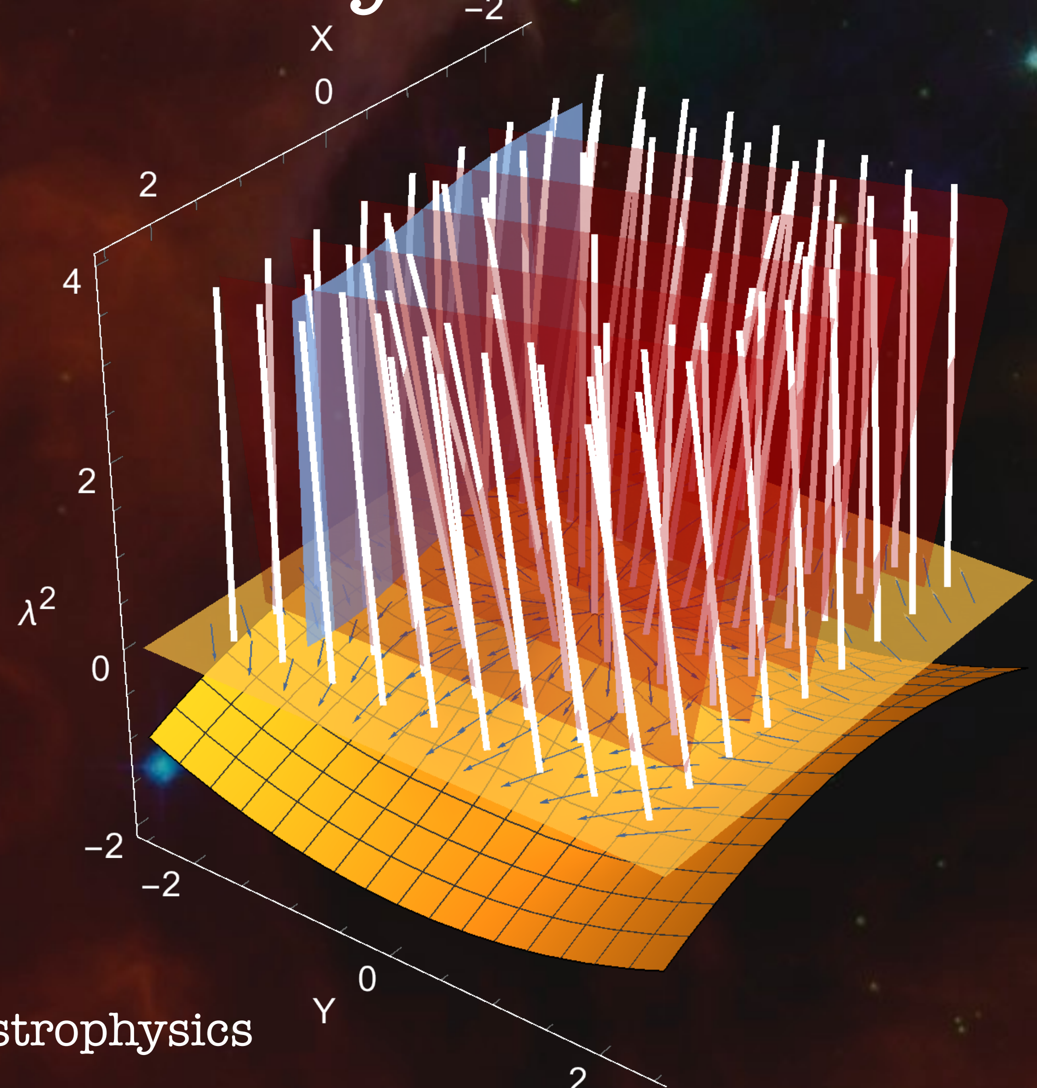
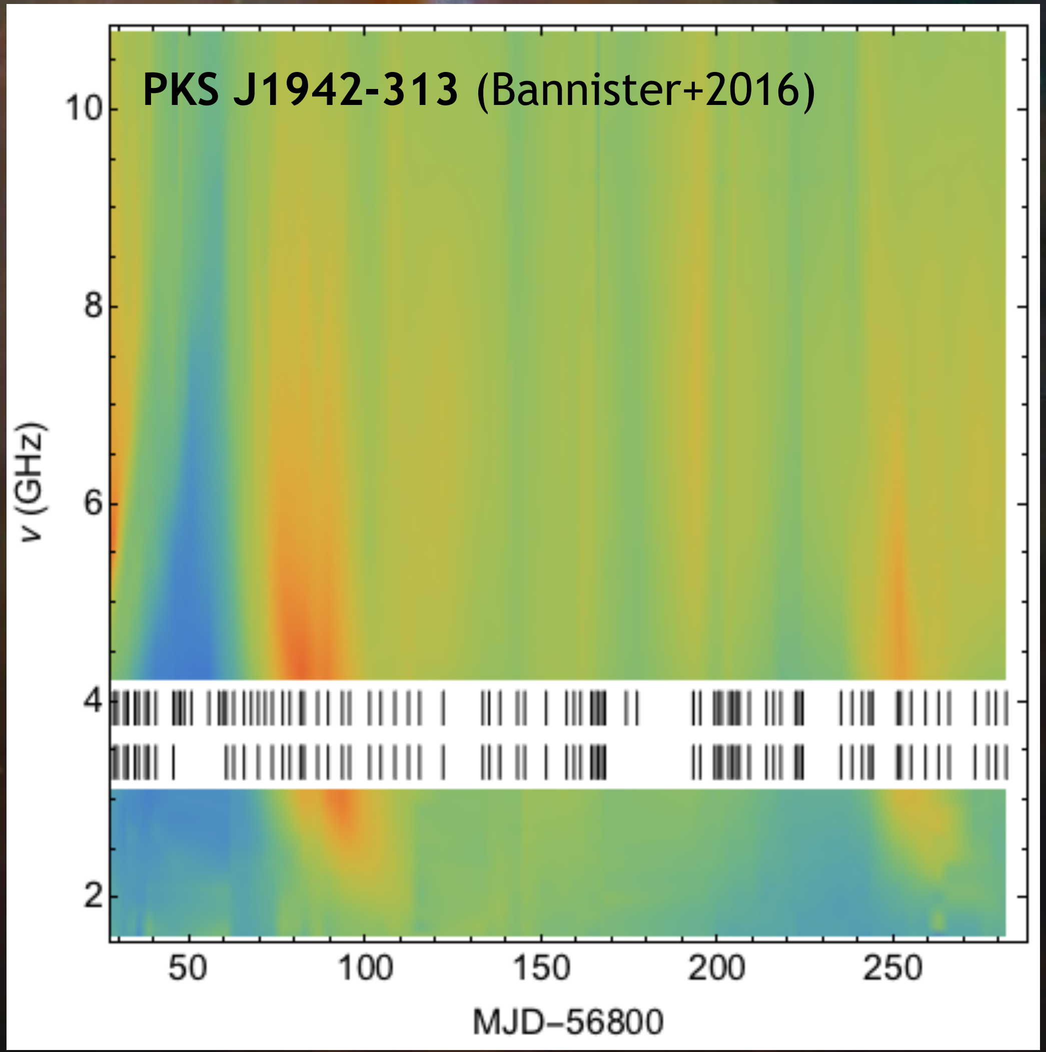
Our solution: use symmetry —
e.g., $N_e(\theta_1,\theta_2) = N_e(\theta_1)$ or $N_e(\boldsymbol{\theta}) = N_e(\theta)$

- sheets of equivalent bristles that cut data cylinder



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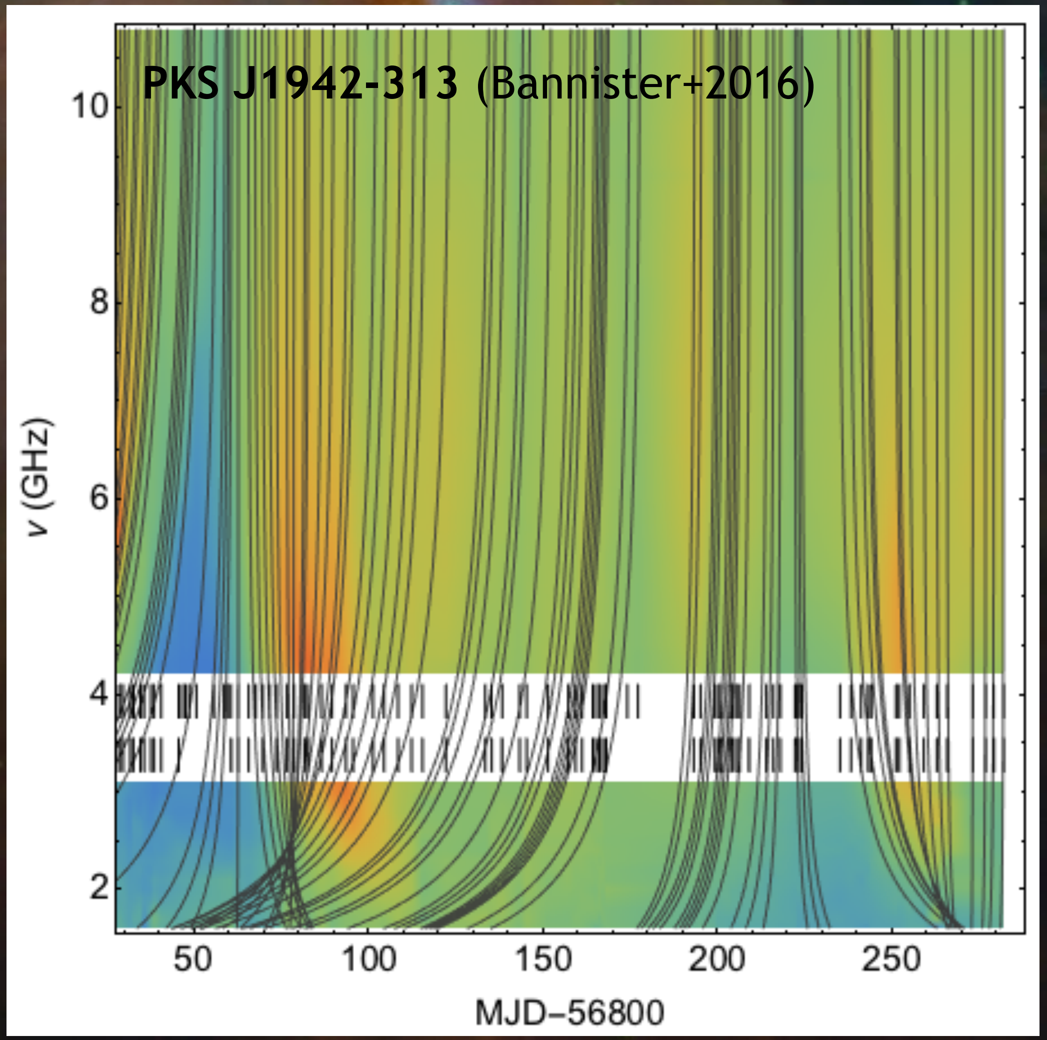
- they are characteristics of $\boldsymbol{\alpha}(\boldsymbol{\theta})$: all points along line probe same $\boldsymbol{\theta}$
- known shape $\beta(\lambda)$, known $\mu(\lambda)$
- go along, reap $\alpha,\kappa,\gamma$, repeat




$$\bar{\beta} = \bar{\theta} - \lambda^2 \bar{\alpha}$$

$$\pm\mu(\lambda)^{-1} = 1 - 2\lambda^2\kappa + \lambda^4(\kappa^2 - \gamma^2)$$

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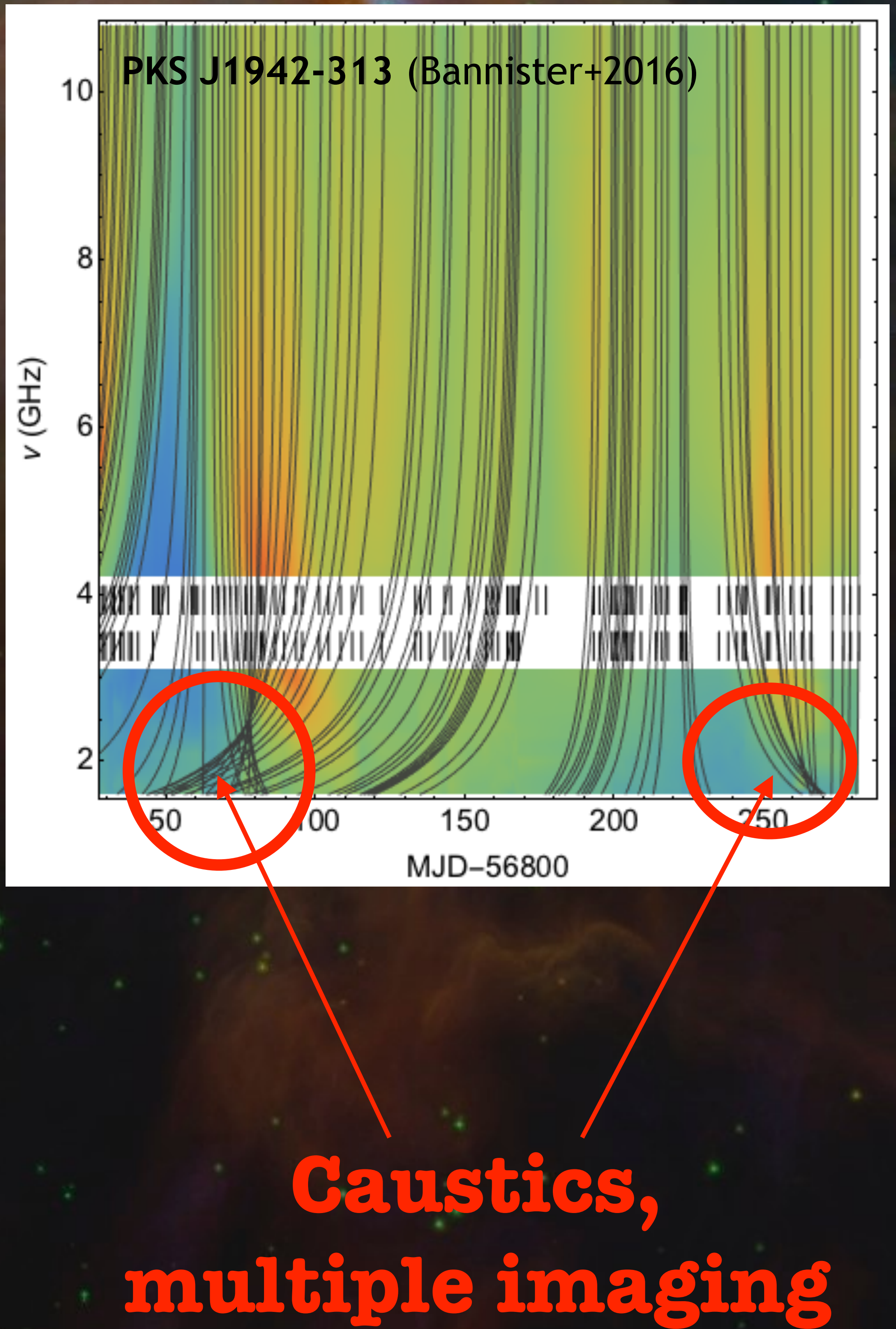
Fails when bristles cross:

- point to different $\boldsymbol{\theta}$, $\mu$ is a sum



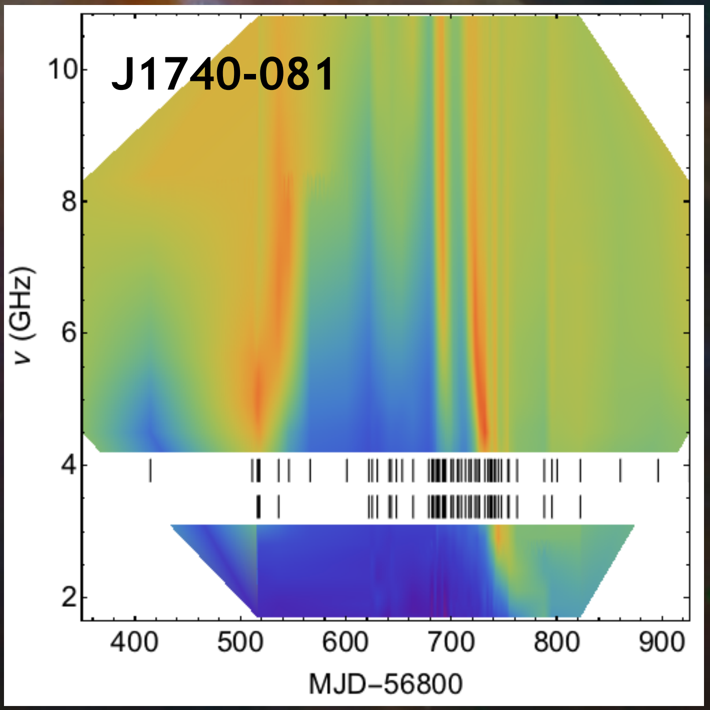
# Caustics: everywhere

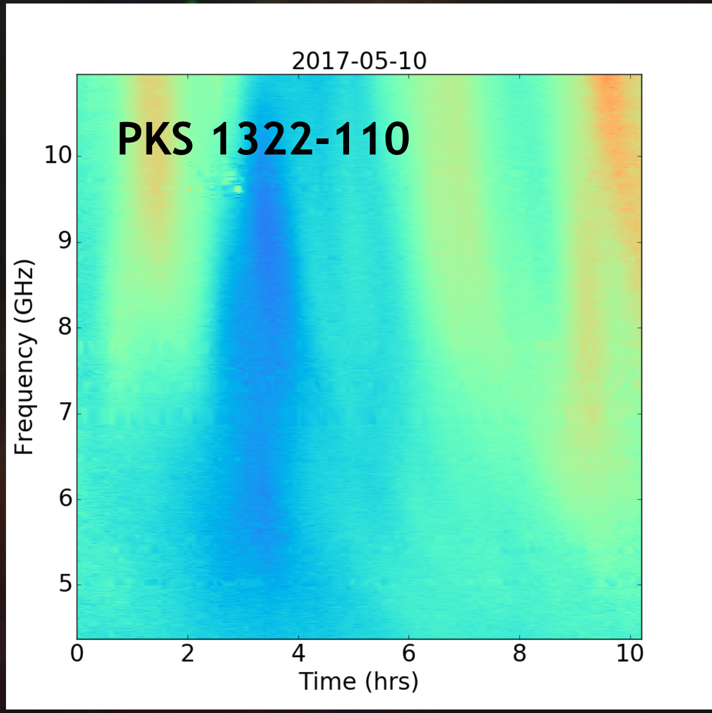
Expected: plasma strengthens as $\lambda^2$

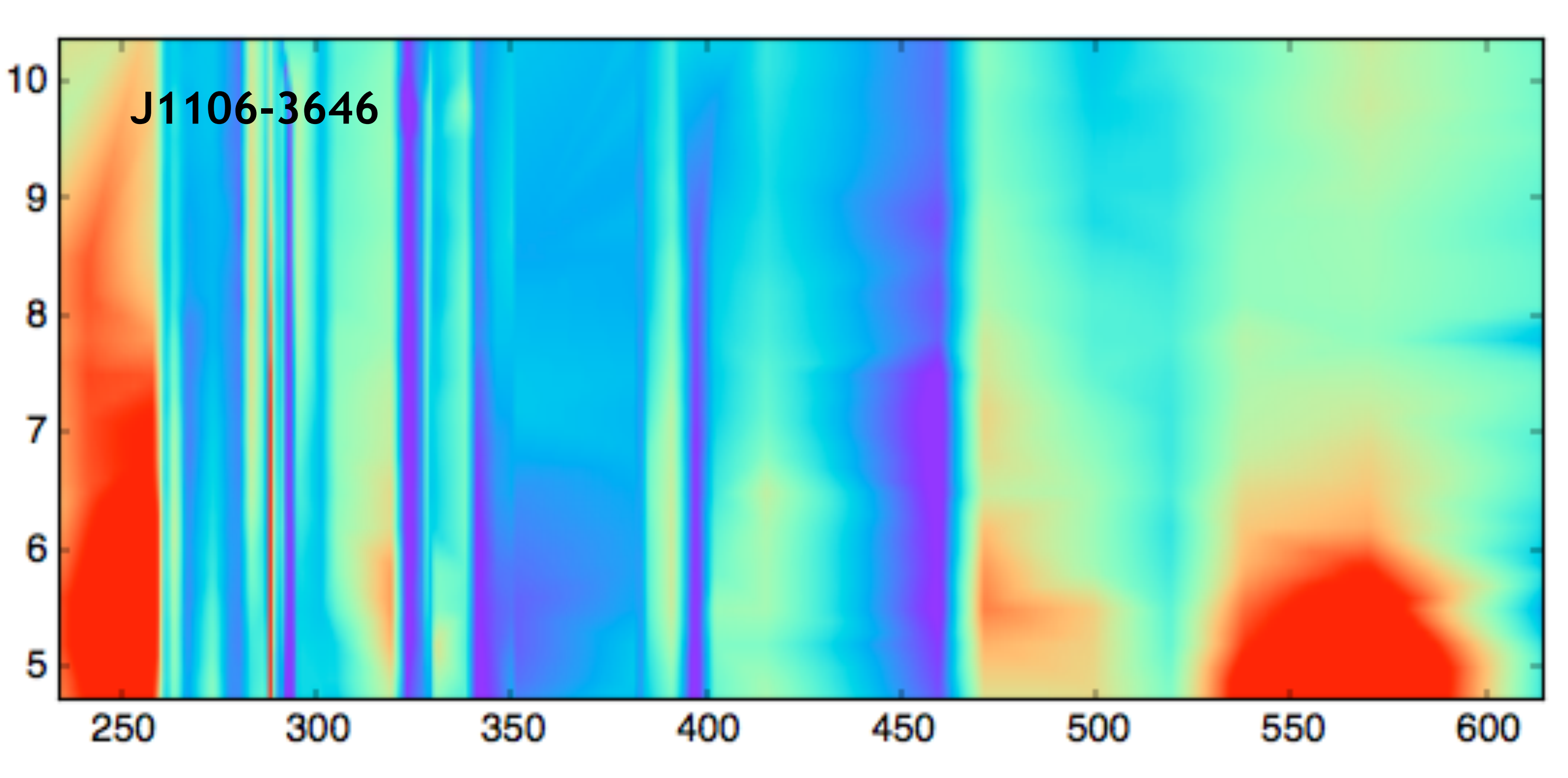
- strong at low enough frequency
- (if not for source size/diffraction)

Ubiquitous: similar patterns in various sources/scales

Useful, sometimes indispensable



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Expected: plasma strengthens as λ²

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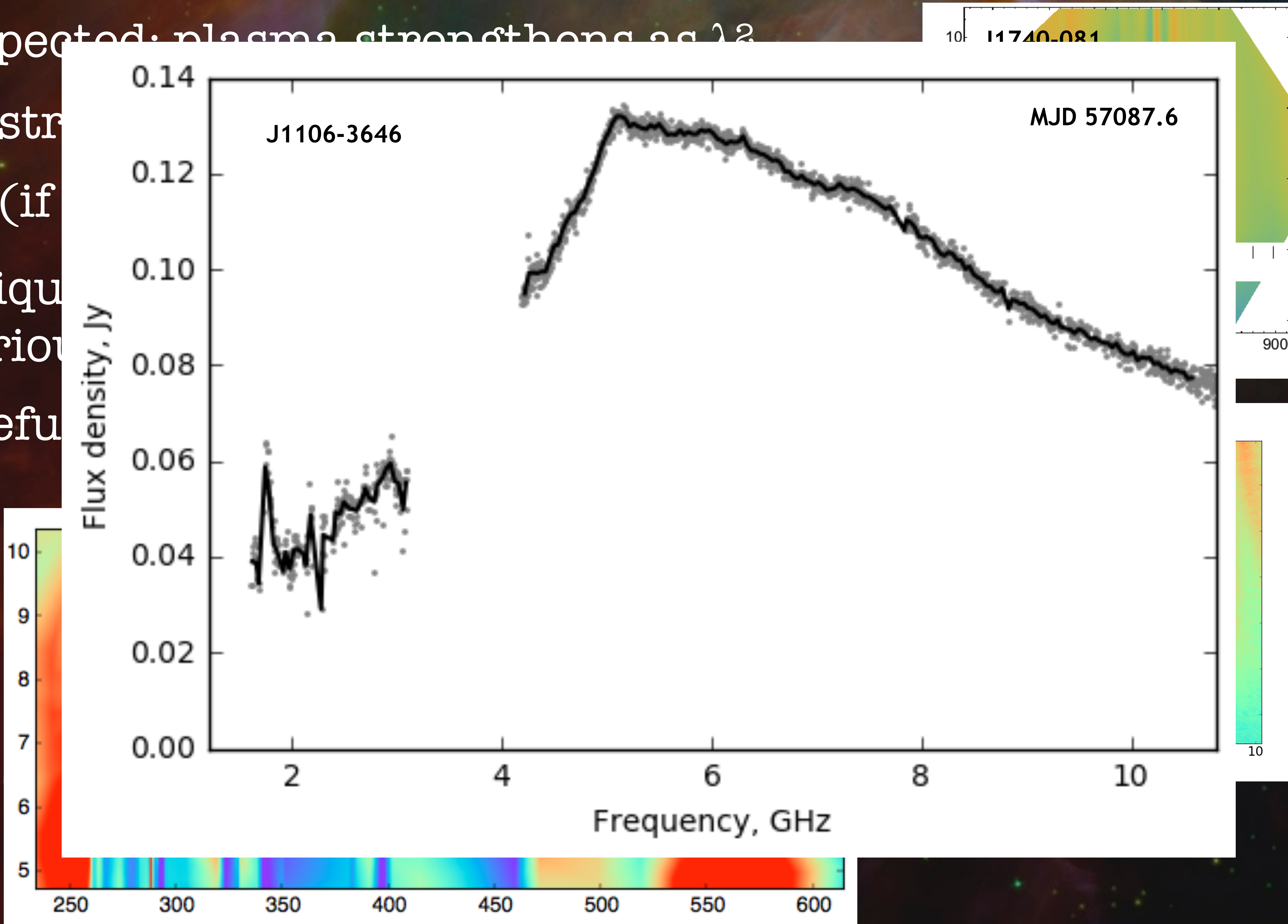

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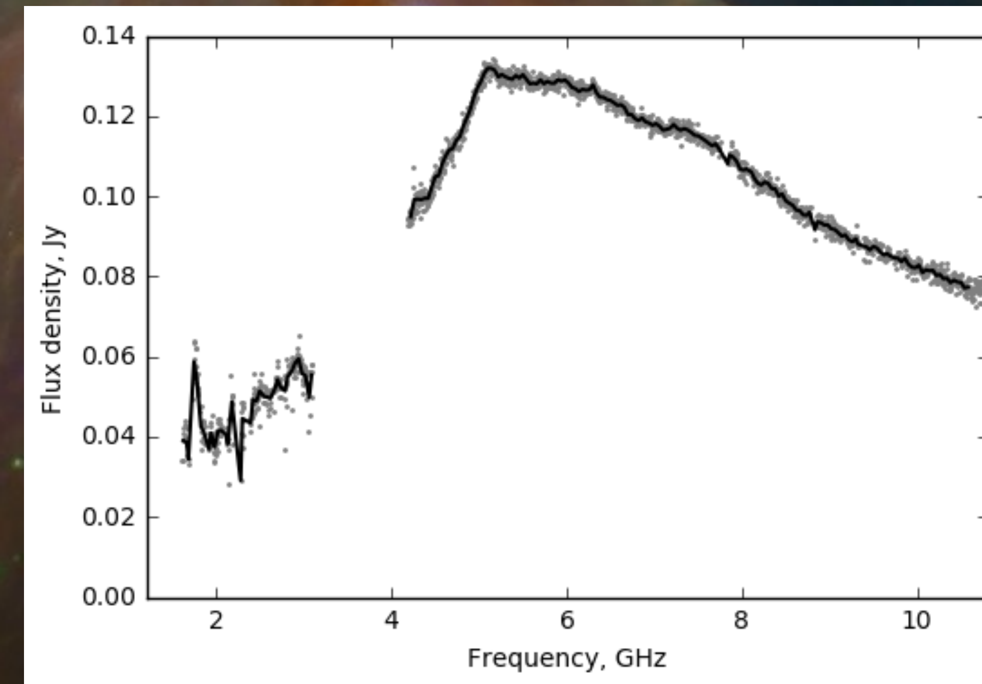


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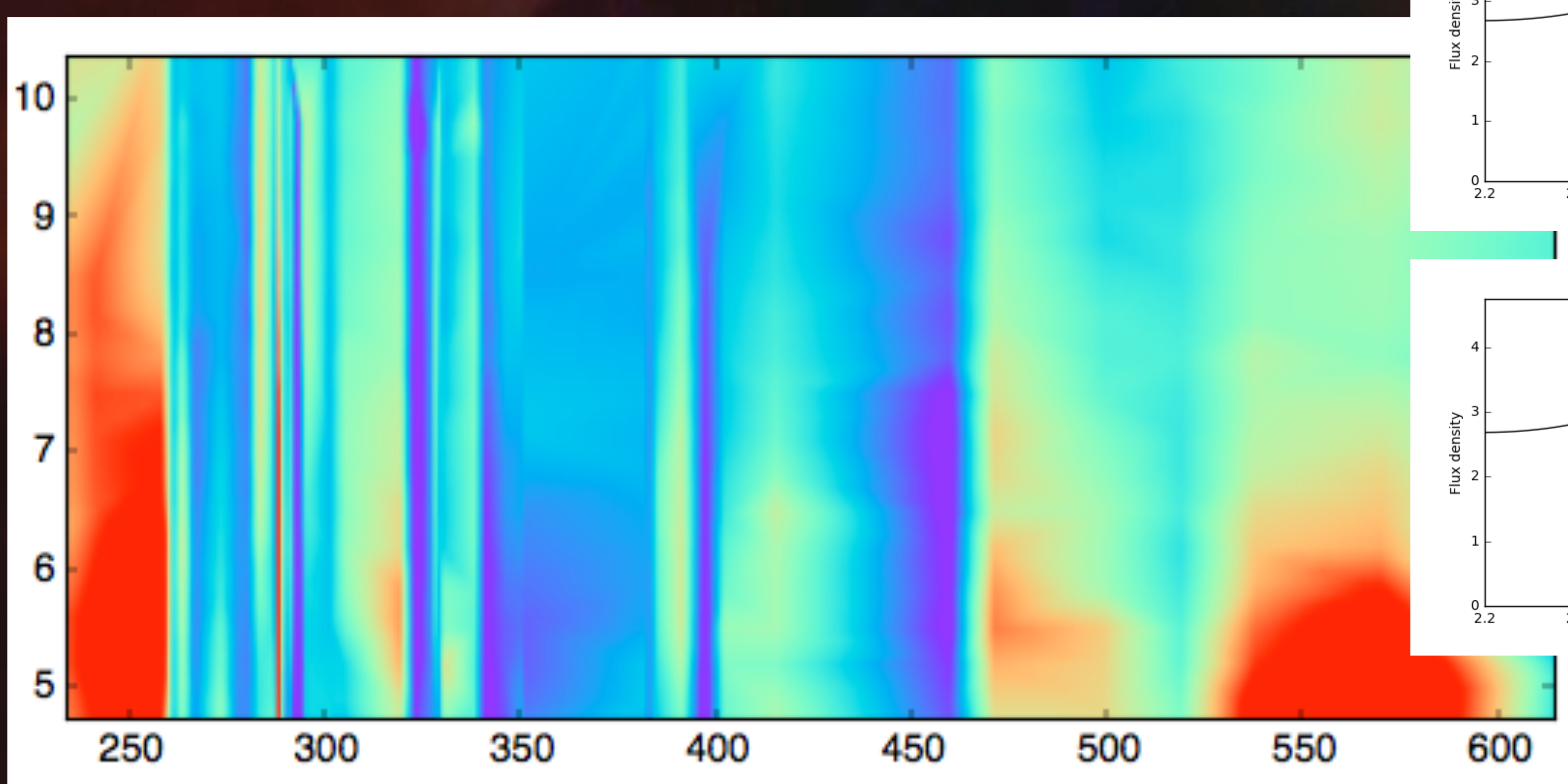
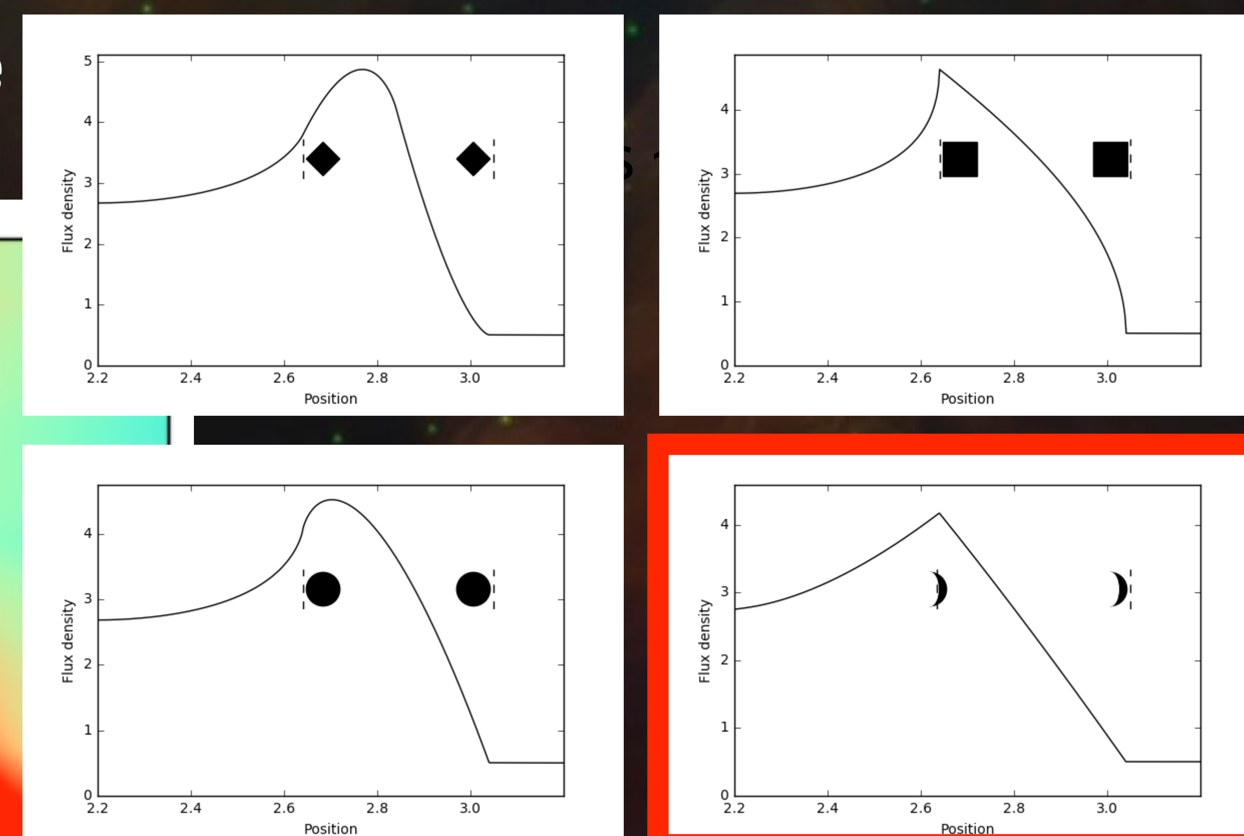



Fold + Concave Source

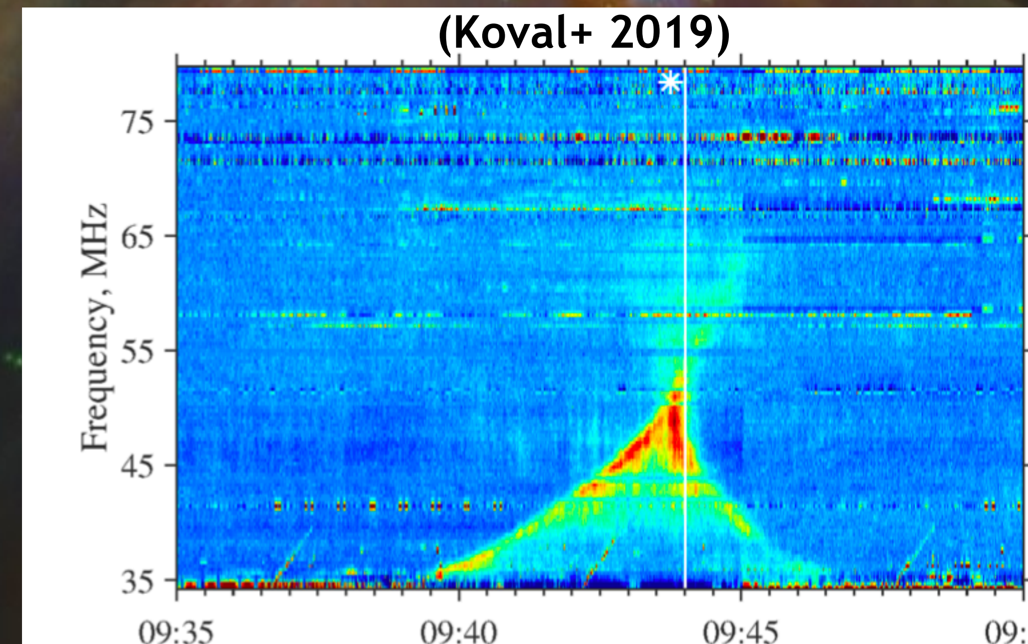
# Caustics: not only interstellar

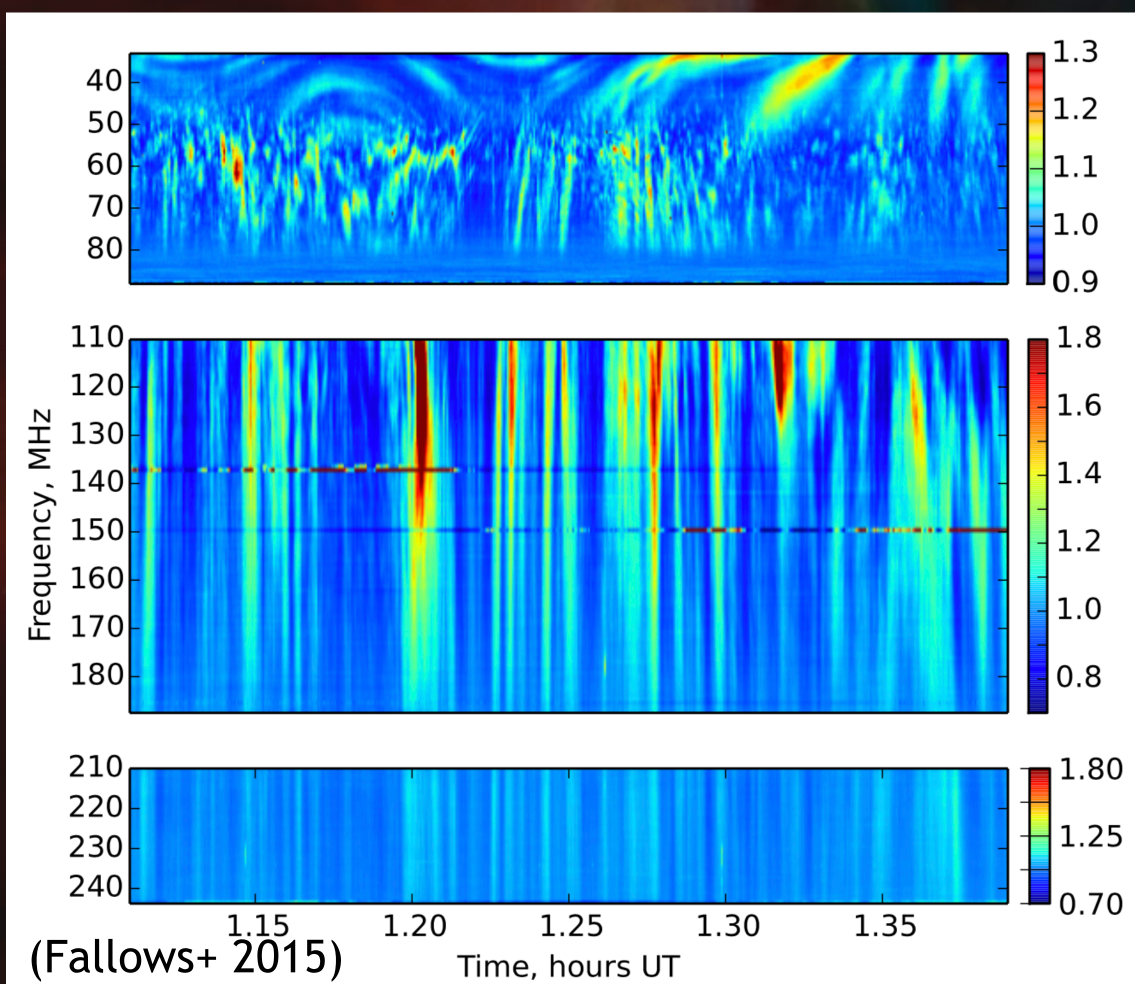
Ionospheric:

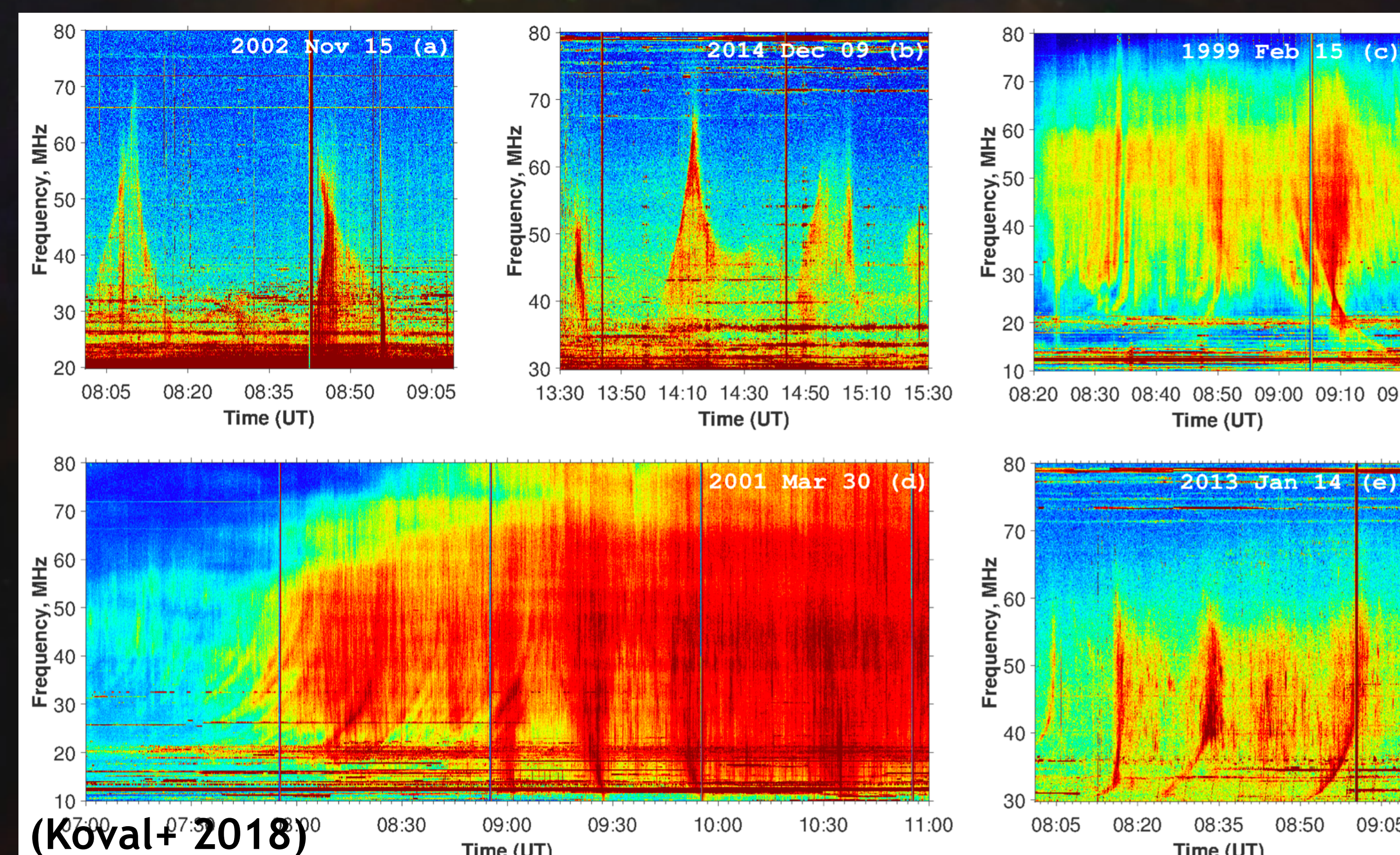
- Fallows+ @ LOFAR
- Koval+ @ Shandong

Interplanetary:

- Chhetri+ @ MWA




Cyg A

Sun

# Spectral caustic in 1D: Definitions

1D lens equation:

$$\beta(\theta) = \theta - \lambda^2 \alpha(\theta)$$

$\theta$ is critical if all its neighbours project to the same $\beta$: $d\beta/d\theta=0$.
Such $\beta$ is caustic.

In 1D, they are points, not curves.

In plasma lensing, each $\theta$ critical:

$$\beta'(\theta) = 1 - \lambda^2 \alpha'(\theta) = 0 \text{ at } \lambda^2 = 1/\alpha'$$

$\Rightarrow$ $\alpha(\theta)$ defines spectral caustic (line):

$$\lambda^2(\theta) = \frac{1}{\alpha'(\theta)}, \qquad \beta(\theta) = \theta - \frac{\alpha(\theta)}{\alpha'(\theta)}$$

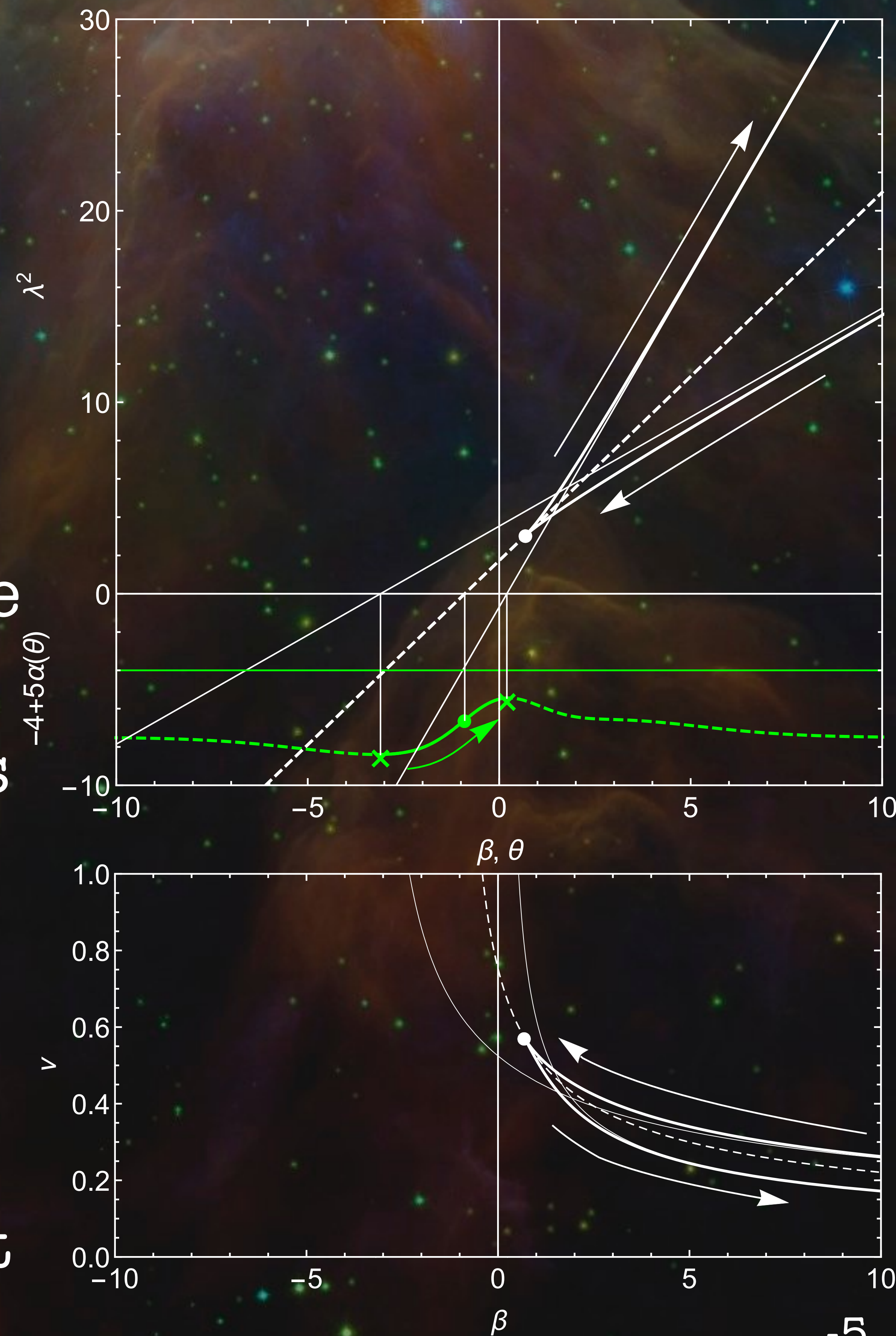
# Spectral caustic in 1D: Anatomy

Spectral caustic:

$$\lambda^2(\theta) = \frac{1}{\alpha'(\theta)}, \quad \beta(\theta) = \theta - \frac{\alpha(\theta)}{\alpha'(\theta)}$$

Only increasing sections of $\alpha(\theta)$ make sense ($\lambda^2>0$), each section draws its caustic, which

- everywhere tangent to a bristle (bristles <u>start</u> crossing here)
- asymptotes to extrema bristles ($\alpha' \to 0$, $\lambda^2=1/\alpha' \to \infty$)
- cusps around inflexion bristles (reaches min $\lambda^2$, turns around)
- does not care of other sections
- measures $\alpha'$ and $\alpha$ at each point



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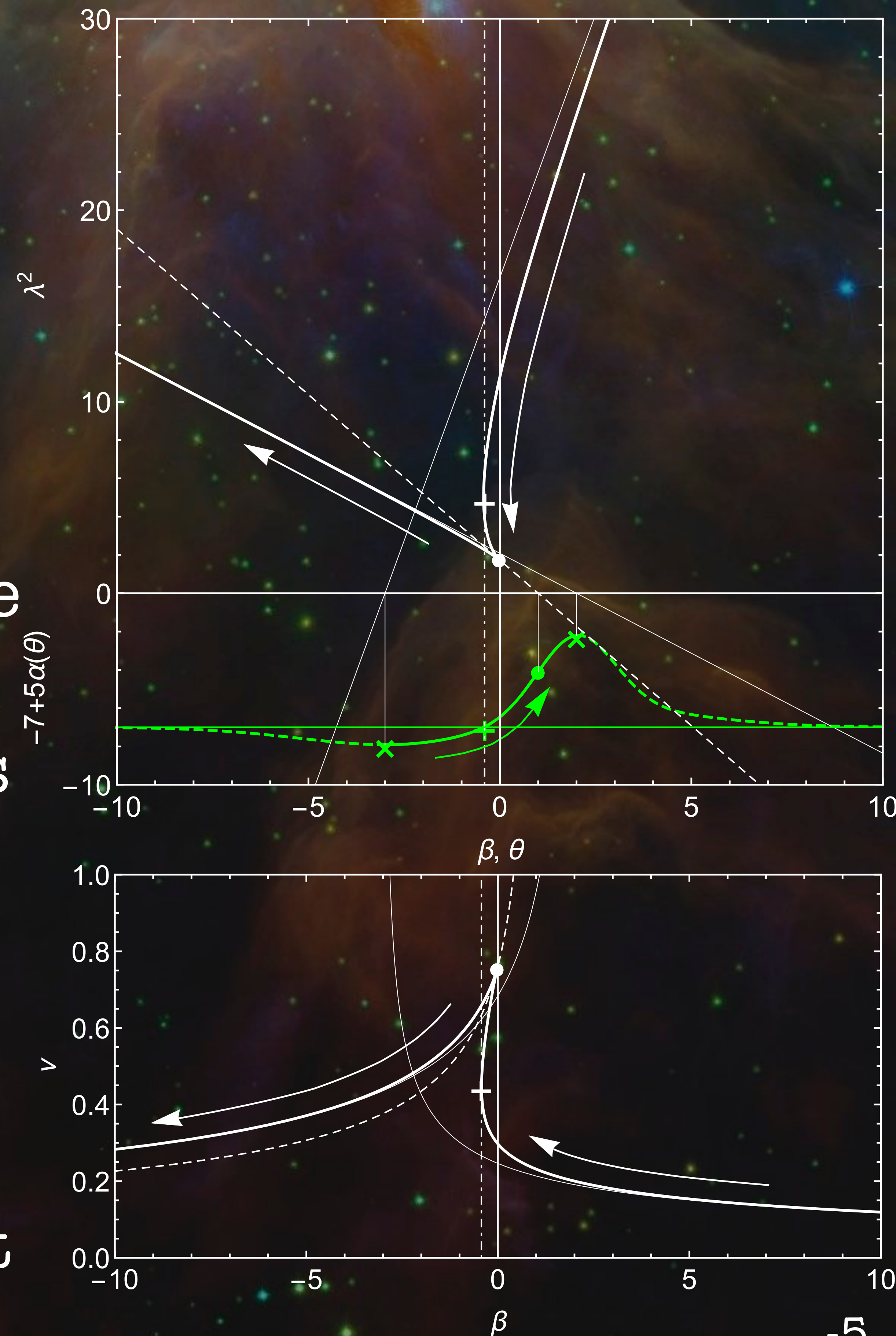

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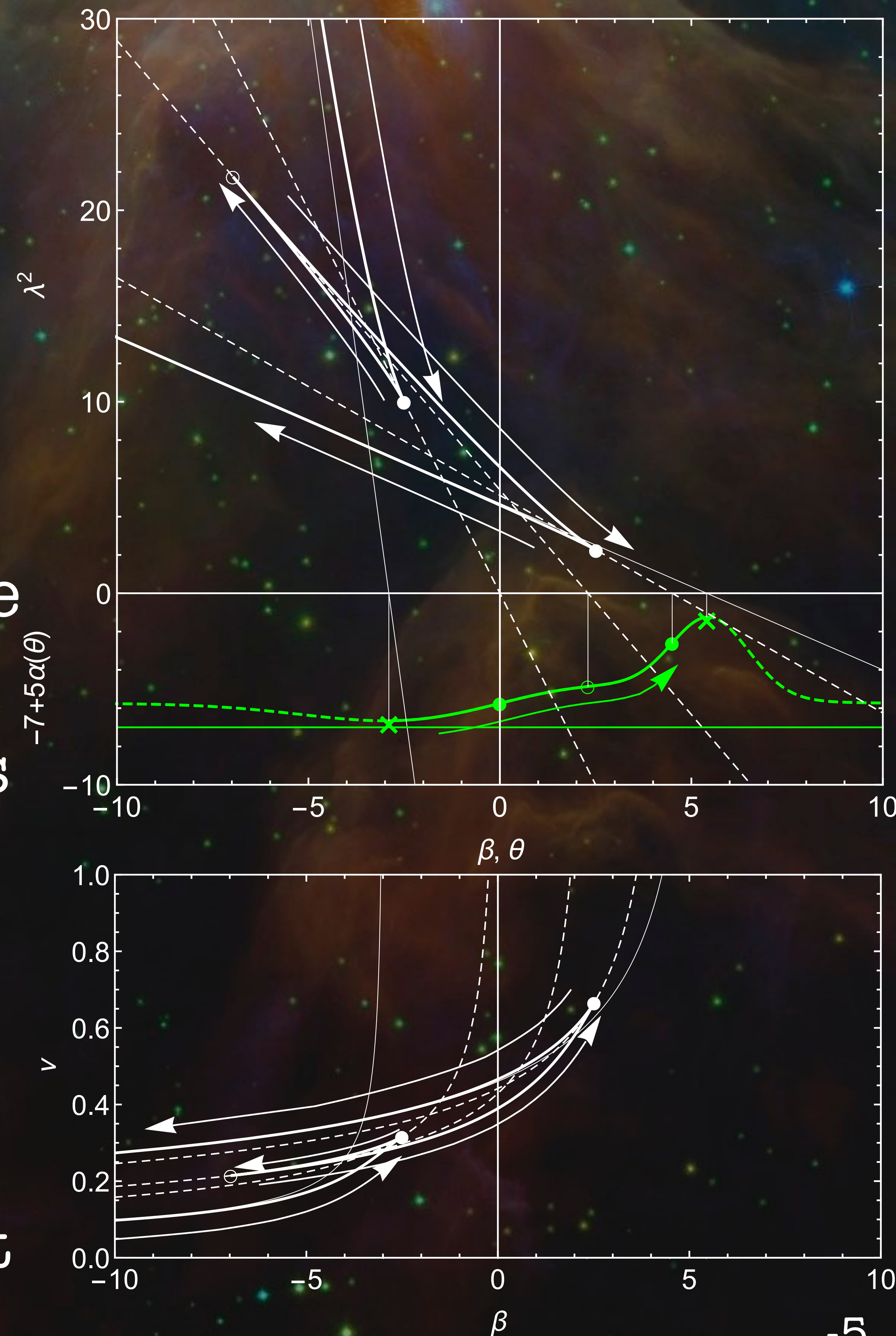

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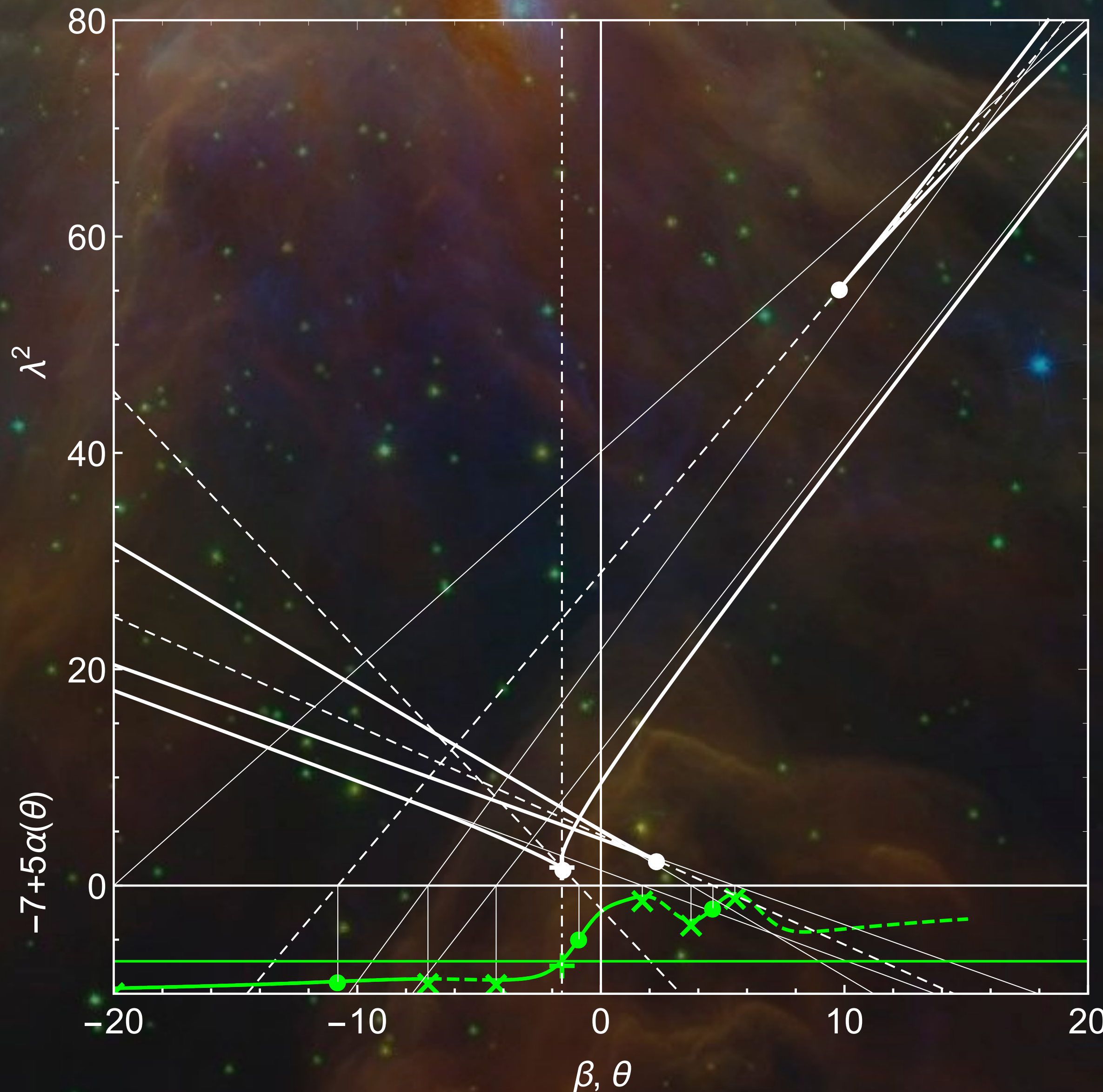
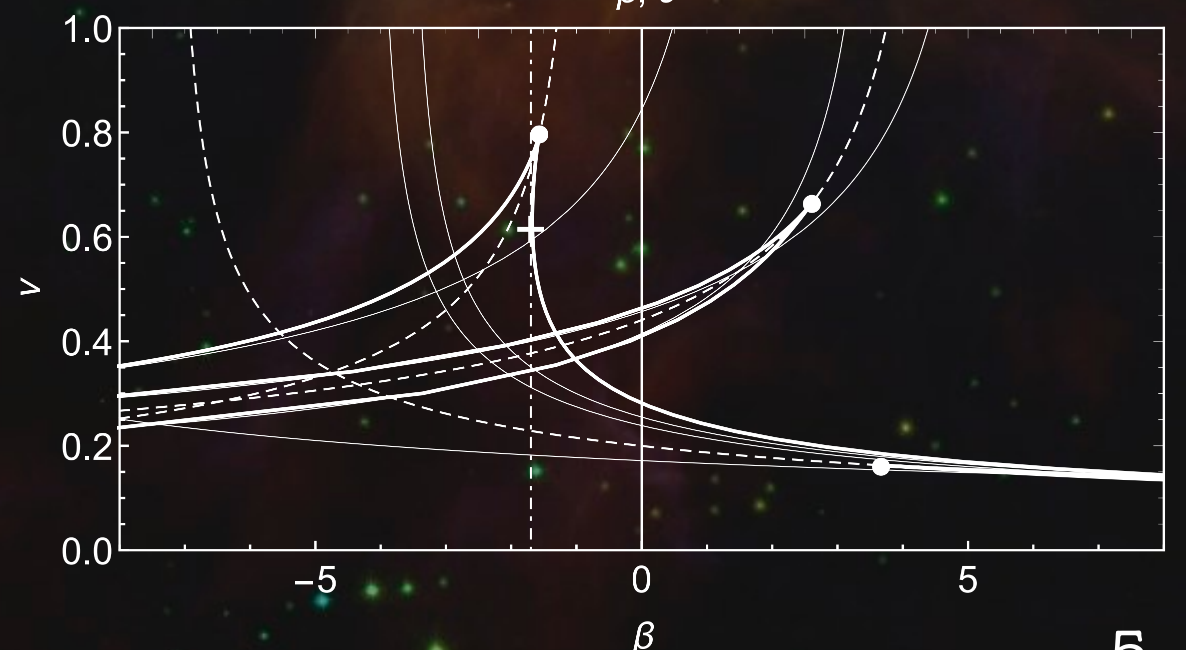
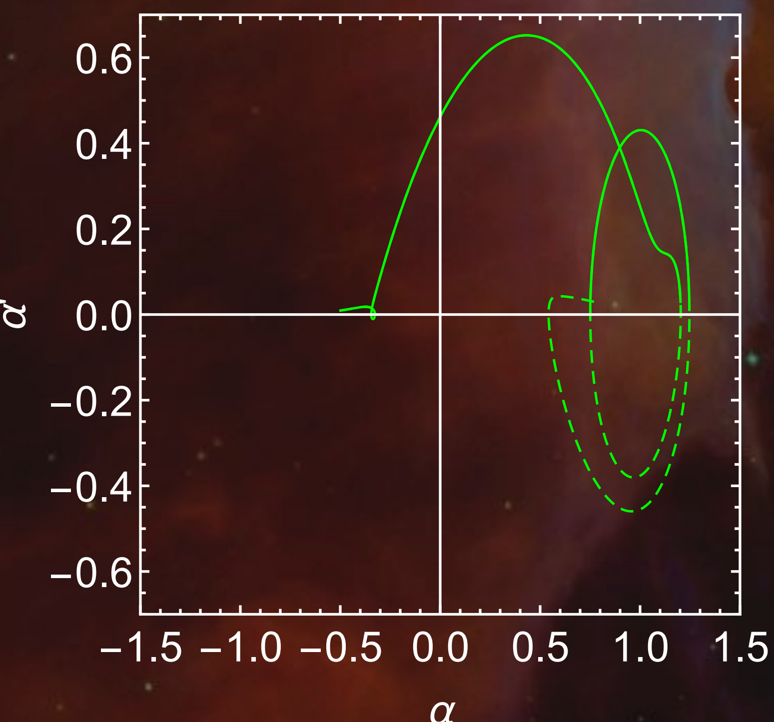


# Spectral caustic in 1D: Gauge

At every point of spectral caustic:

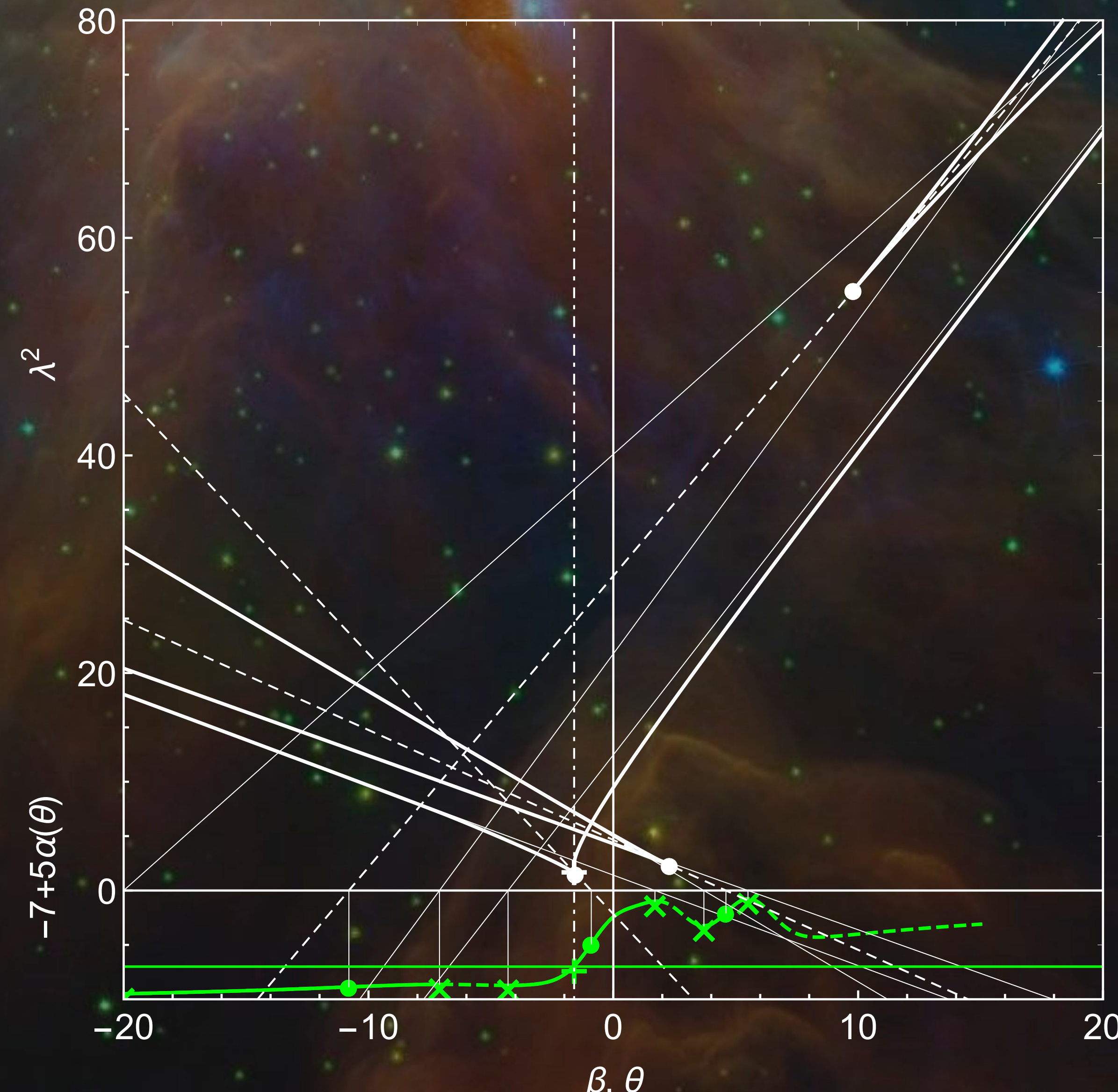
$$\alpha = -\frac{d\beta}{d\lambda^2}, \quad \alpha' = \lambda^{-2}$$




$$\Rightarrow \alpha(\theta + C_i)$$

at each section.

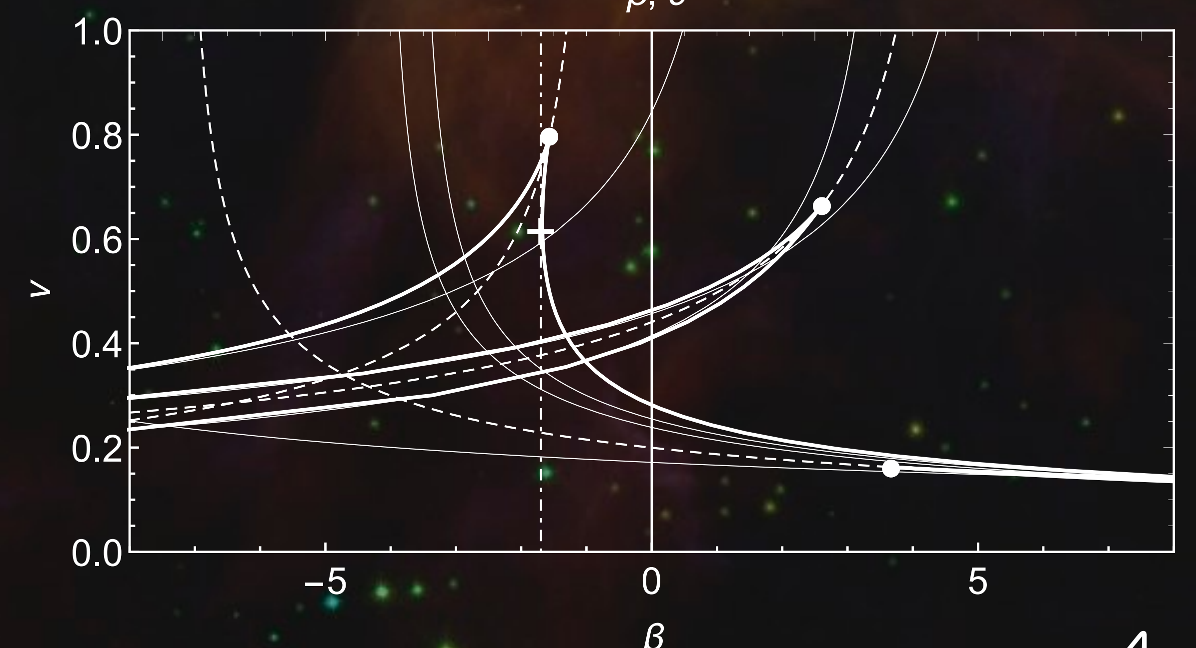
Phase portrait curl statistics?




Works for other parametrisations.
For density in physical units:

$$N_e'' = \frac{\nu_f^2}{D_{\rm eff}\, r_e c^2}, \; N_e' = \frac{V_{\rm eff}\, \nu_f^3}{2 D_{\rm eff}\, c^2 r_e} \frac{dt_f}{d\nu_f}$$

need screen distance and velocity.



# Spectral caustics in 2D

2D lens equation: $\bar{\beta}(\bar{\theta}) = \bar{\theta} - \lambda^2\,\bar{\alpha}(\bar{\theta})$

Locally, $\delta\bar{\beta} = \hat{A}\delta\bar{\theta}, \quad A_{ij} = \delta_{ij} - \lambda^2 \dfrac{\partial \alpha_i}{\partial \theta_j}$

***α(θ)*** is a gradient, so **A** is symmetric:

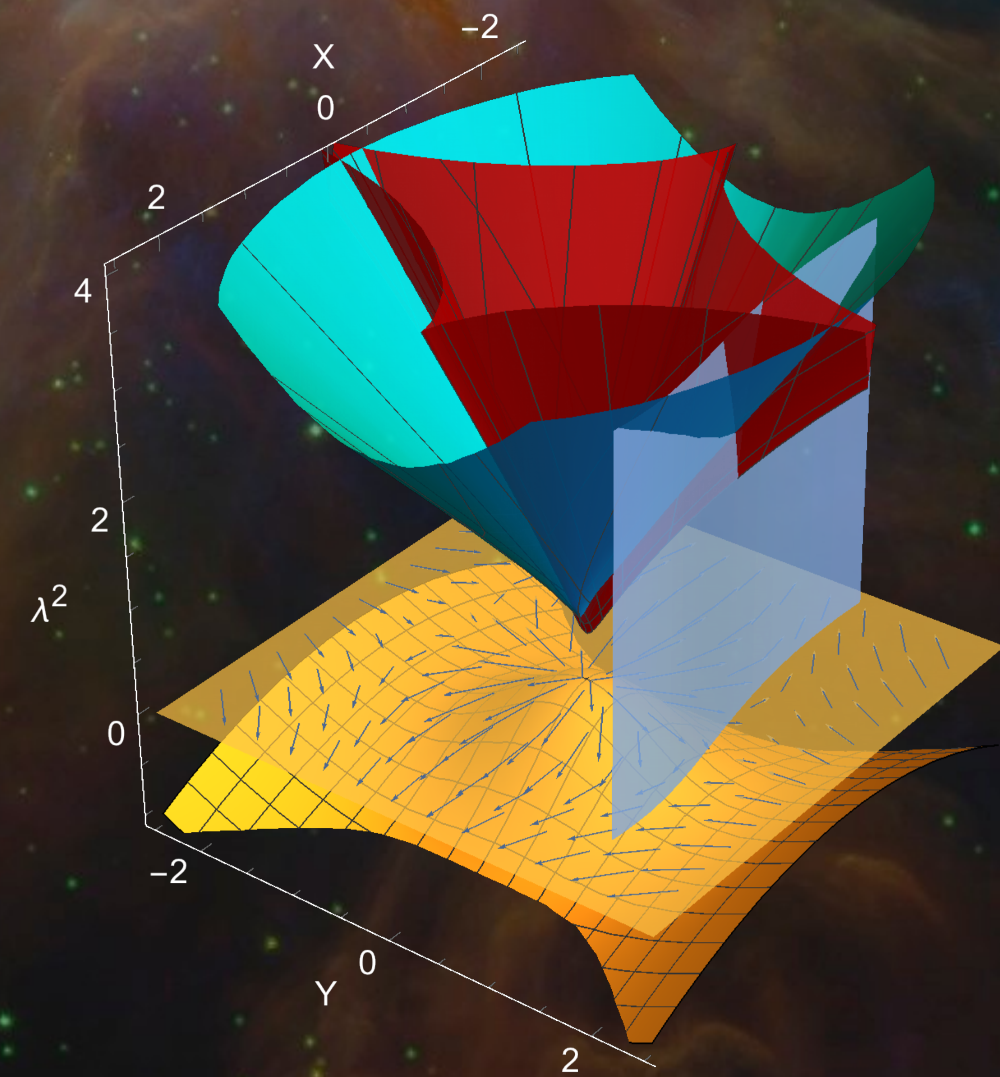
$$\hat{A} = (1-\lambda^2\kappa)\hat{1} - \lambda^2\gamma \begin{pmatrix} \cos 2\chi & \sin 2\chi \\ \sin 2\chi & -\cos 2\chi \end{pmatrix}$$

***θ*** is critical when **A** is degenerate:

$$\det \hat{A} = \left[1 - \lambda^2(\kappa+\gamma)\right]\left[1 - \lambda^2(\kappa-\gamma)\right] = 0$$

$\Rightarrow$ Two spectral caustics surfaces:

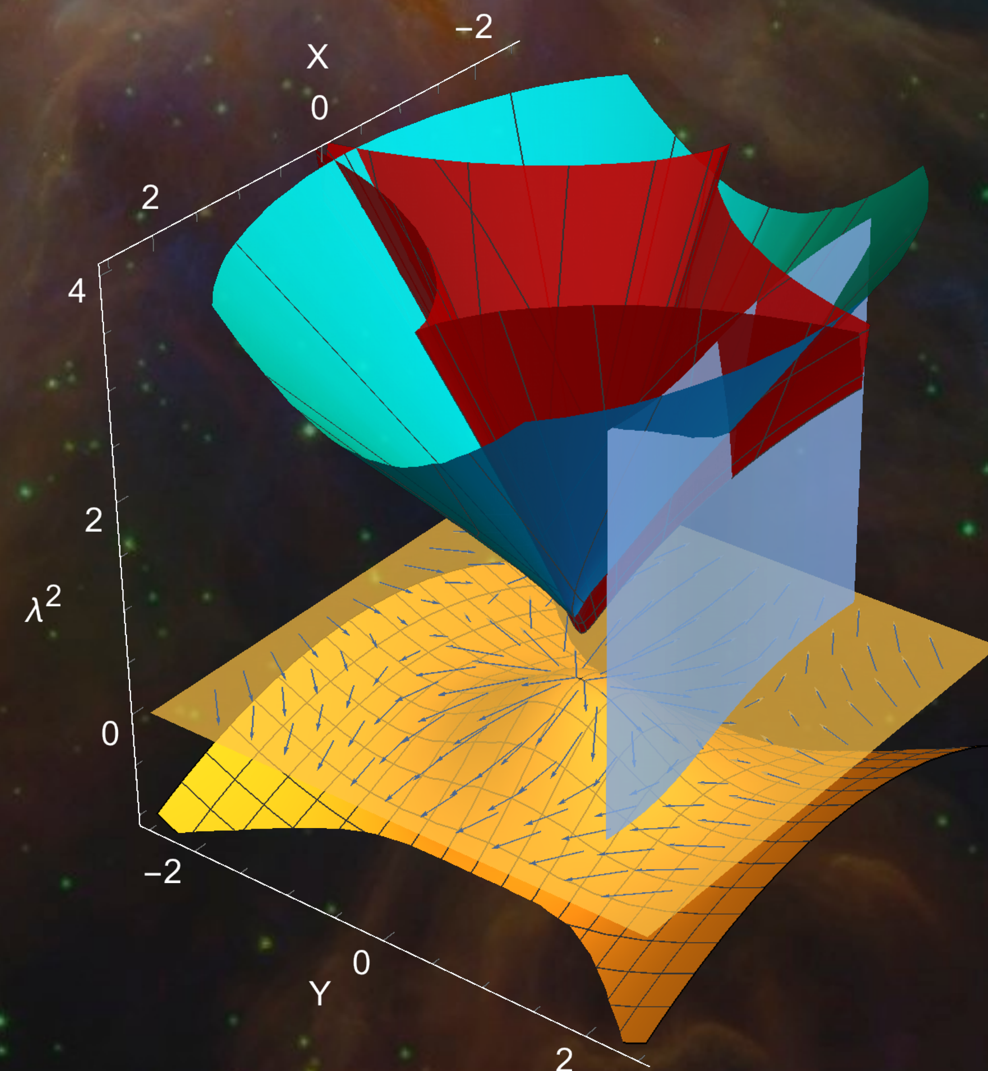
$$\lambda^2(\bar{\theta}) = \frac{1}{\kappa(\bar{\theta}) \pm \gamma(\bar{\theta})}, \quad \bar{\beta}(\theta) = \bar{\theta} - \frac{\bar{\alpha}(\theta)}{\kappa(\bar{\theta}) \pm \gamma(\bar{\theta})}$$




Spectral caustics are intersections with data cylinder.

# Spectral caustics in 2D: surface properties

Two independent “big caustics”, ± , crossing above (at $\kappa^{-1}\alpha$) $\gamma$=0 line




Locally, like 1D spectral caustics:

- tangent to bristles at each point
- break into sections asymptoting to cones on lines: $\kappa=\mp\gamma$, ($\kappa$ can be <0)
- sections can cut each of other
- normal vectors our friends (directions, angles, intersections)

Globally, need catastrophe theory:

- sections consist of smooth leaves ⬅ folds
- joined at “bicaustic line” creases ⬅ cusps
- cusp patterns morph above $\delta(\kappa \pm \gamma)=0$ ⬅ lips/beak-to-beak
  - $\gamma=0$ line is a metamorphose, too ⬅ hyperbolic umbilic

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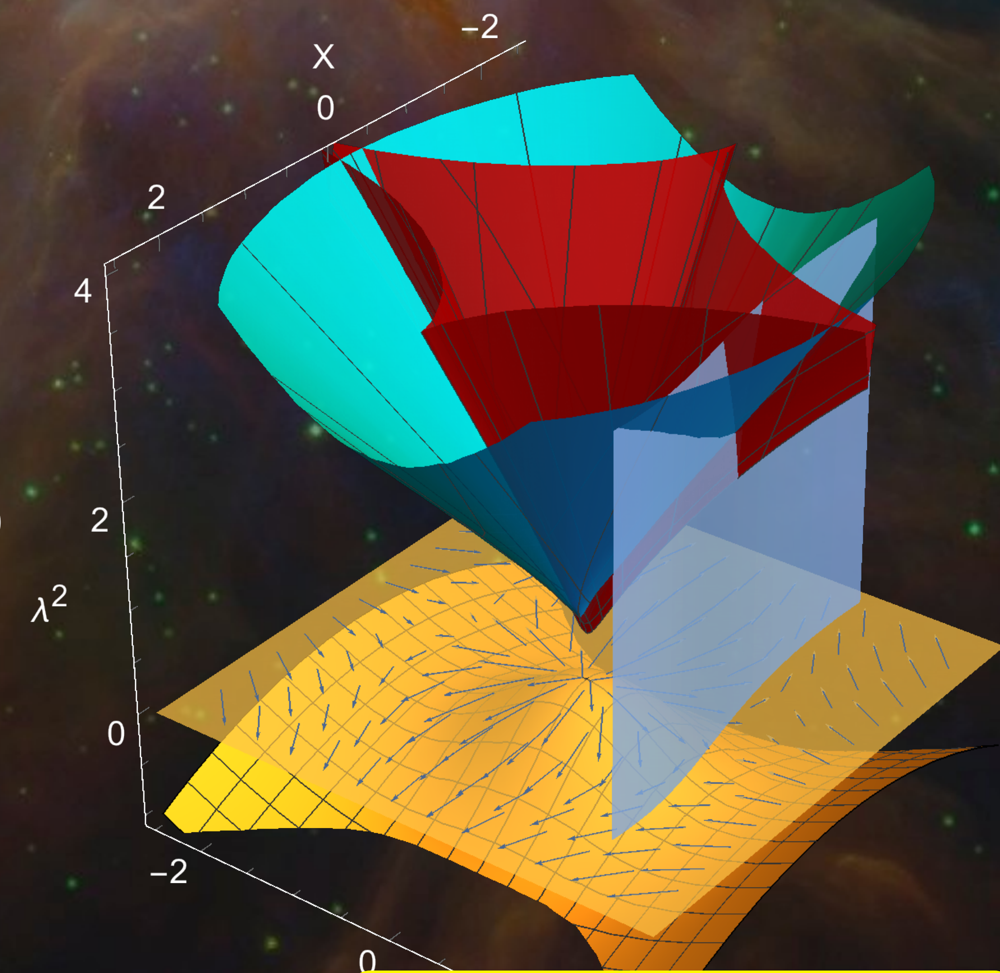


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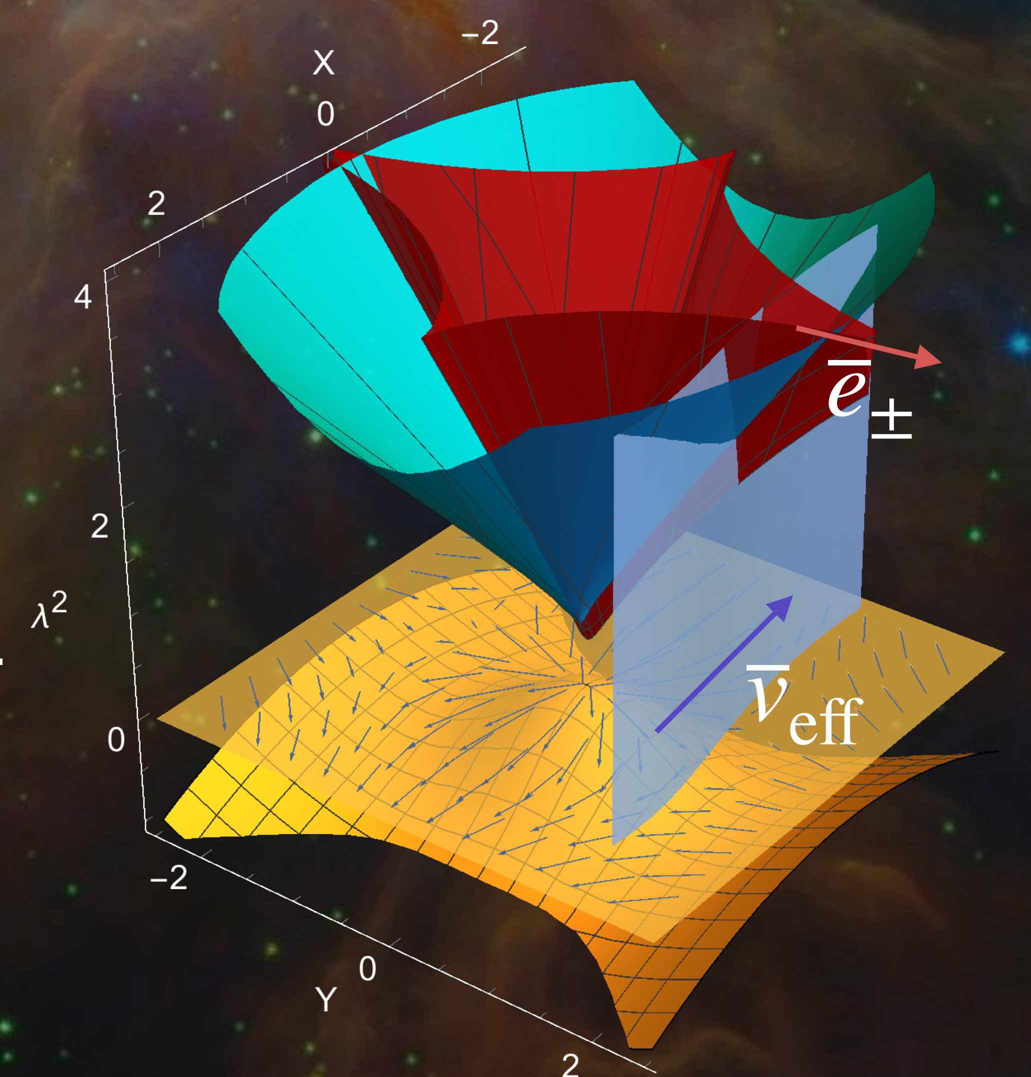
But two, ±, big caustics can cross "accidentally", too, at $\gamma\neq0$

# Spectral caustics in 2D: data projection

Spectral caustics are big caustic intersection with data cylinder.

- folds cut data along smooth lines
  - can only end at data edges
  - or break at cusps
- where smooth $\dfrac{\mathrm{d}t}{\mathrm{d}\lambda^2} = -\dfrac{D_{\mathrm{eff}}\,(\overline{\alpha}\cdot\overline{e}_{\pm})}{(\overline{v}_{\mathrm{eff}}\cdot\overline{e}_{\pm})}$

  ($\boldsymbol{e}_{\pm}$ is local orientation of shear)
  - can run parallel to either axis
  - bounds, but does NOT measure $\boldsymbol{\alpha}$
    - varying $\boldsymbol{v}_{\mathrm{eff}}$ direction can help
  - phase portraits can still be useful
- $N_e'' = \dfrac{\nu_f^2}{D_{\mathrm{eff}}\, r_e c^2}$ holds



# Summary & Outlook

- Spectral caustics are prominent in dynamic spectra
- In high anisotropy (1D):
  - all same shape
  - directly measure $N_e(\theta)$
- In full 2D:
  - have complex, though recognisable, morphologies
  - only measure principal curvatures of $N_e(\boldsymbol{\theta})$
  - bound, but cannot measure its gradient
- Can we do better?
  - multiple cuts (think repeating FRBs, two+ stations)
  - time domain: universal flux ratios/inversions